4F5: Advanced Communications and Coding

Handout 8: Modelling the Wireless Channel

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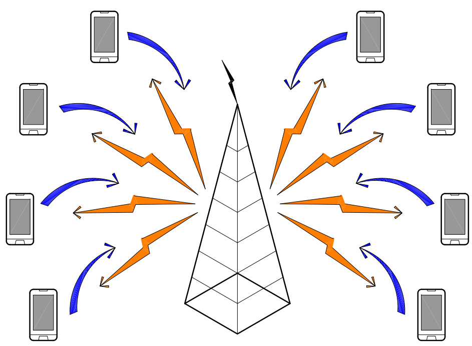
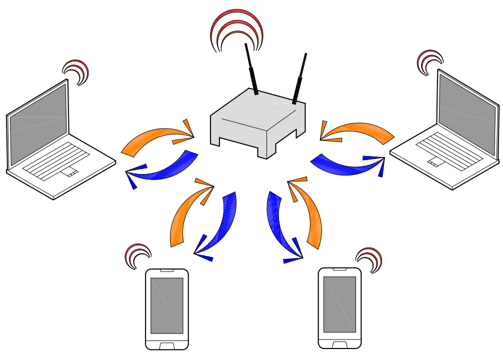
The Wireless Channel

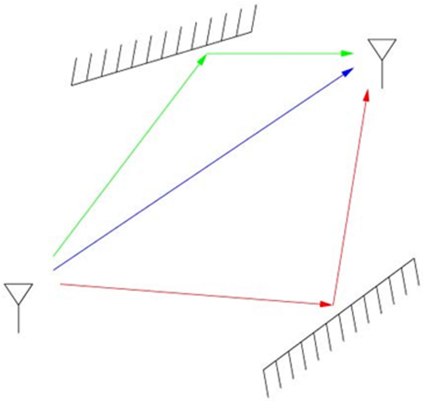


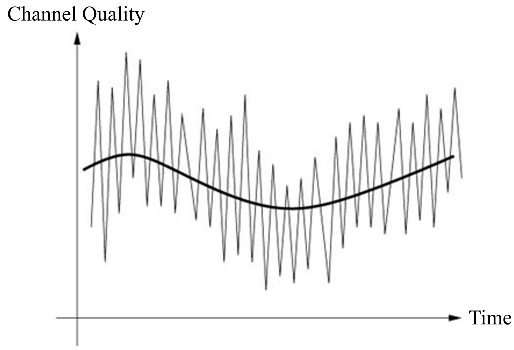


Two fundamental aspects of wireless communication:

1. *Fading*: Time variation of channel strengths
2. *Interference* between various users. For example,
   - Between several transmitters communicating with a common receiver (e.g. mobiles to base-station)
   - Between single transmitter and multiple receivers

Here we'll only consider *fading* and study how it affects system design



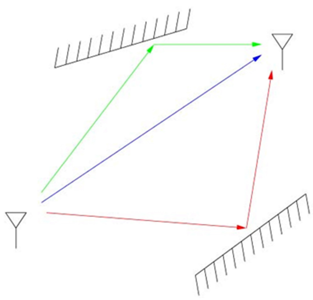


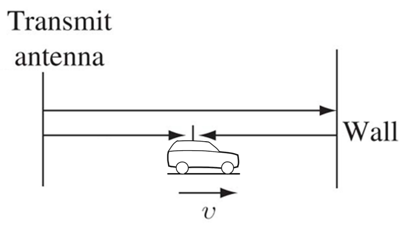
From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

The channel quality varies over two time-scales:

1. *Large-scale* fading: Occurs at a slow time scale
   - Path loss of signal as function of distance and shadowing by large objects such as buildings, hills
   - Time constants associated with variations are of the order of many seconds or minutes
   - More important for cell site planning, less for Tx/Rx design
2. *Small-scale* fading: Occurs at a fast time scale
   - Due to constructive and destructive interference of the multiple signal paths between the Tx and Rx
   - Occurs as mobile moves at the spatial scale of order of carrier wavelength, and is frequency dependent

Small-scale Multipath Fading



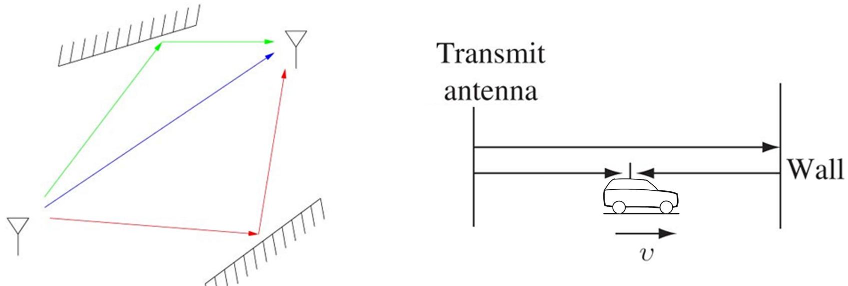


From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

- Multipath fading due to *constructive* and *destructive* interference of transmitted waves
- Carrier frequency f_c for wireless communication is of the order of GHz. E.g. For cellular, $f_c = 0.9$ GHz or 1.9 GHz
- Channel varies when mobile moves a distance of the order of the carrier wavelength $\lambda_c = c/f_c$ (about 0.3m for cellular)
- For vehicular speeds, this translates to channel variation of the order of ~ 100 Hz

How do parameters such as f_c, bandwidth, mobile speed, delay spread affect communication system design

Physical Model



From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

The wireless channel can be modelled as

$$Y(t) = \sum_i a_i(t)\, X(t - \tau_i(t)) + N(t)$$

- On path i from the transmitter to receiver:
  $a_i(t)$ is the attenuation *at time t*
  $\tau_i(t)$ is the propagation delay *at time t*
- The sum is over all *paths i*
- This is a linear, *time-varying* system

Consider first the special case where the channel is time-invariant, i.e., $a_i(t) = a_i$ and $\tau_i(t) = \tau_i$ for all i

Time-invariant Fading Channel

$$Y(t) = \sum_i a_i X(t - \tau_i) + N(t) \tag{1}$$

For concreteness, assume that $X(t)$ is the foll. QAM signal:

$$X(t) = \sum_k p(t - kT)\ [\mathrm{Re}(X_k)\cos(2\pi f_c t)\ -\ \mathrm{Im}(X_k)\sin(2\pi f_c t)]$$

Let us write (1) in terms of the *baseband* equivalent: we want $X_B(T)$ such that

$$X(t) = \mathrm{Re}\Big[\ X_B(t)e^{j2\pi f_c t}\ \Big]$$

$X_B(t)$ is seen to be

$$X_B(t) = \sum_k p(t-kT)[\mathrm{Re}(X[k]) + j\,\mathrm{Im}(X[k])] = \sum_k p(t-kT)X[k]$$

where the symbol time $T \approx \frac{1}{2W}$. Note that $X_B(t)$ is bandlimited to $[-W, W]$.

We have written

$$X(t) = \text{Re}\Big[\; X_B(t) e^{j2\pi f_c t} \;\Big]$$

$$\text{Hence} \quad X(t-\tau_i) = \text{Re}\Big[\; X_B(t-\tau_i) e^{-j2\pi f_c \tau_i} e^{j2\pi f_c t} \;\Big]$$

Let $\tilde{N}(t) = N(t)e^{-j2\pi f_c t}$ be the baseband equivalent of the noise so that

$$N(t) = \text{Re}[\tilde{N}(t)e^{j2\pi f_c t}]$$

and

$$\begin{aligned} Y(t) &= \sum_i a_i X(t-\tau_i) + N(t) \\ &= \text{Re}\Big[\; \Big(\sum_i a_i e^{-j2\pi f_c \tau_i} X_B(t-\tau_i) + \tilde{N}(t) \Big) \cdot e^{j2\pi f_c t} \Big] \end{aligned}$$

Let's define $a_{B,i} = a_i e^{-j2\pi f_c \tau_i}$. Then ...

Towards a Discrete-Time Representation

The equivalent baseband channel is:

$$Y_B(t) = \sum_i a_{B,i}\, X_B(t-\tau_i) + \tilde{N}(t)$$

where $a_{B,i} = a_i e^{-j2\pi f_c \tau_i}$.

Recall $X_B(t) = \sum_k X[k]\; p(t-kT)$ with

$$p(mT) = 1 \text{ for } m = 0, \text{ and } 0 \text{ for } m \neq 0$$

- This sampling property of $p(t)$ is clearly true for sinc or rect.
- In fact, it also holds for the raised cosine pulse, and more generally whenever $p(t)$ satisfies something called the "Nyquist criterion". (3F4 talks about this in detail.)
- For such $p(t)$, symbols can be recovered from $X_B(t)$ can be done by just sampling the output at instants $\{T, 2T, \ldots\}$

- Key Q: What do you get by sampling $Y_B(t)$ at $t = T, 2T, \ldots$
- Which $X[k]$'s contribute to $Y_B(mT)$ for $m = 1, 2, \ldots$?

A Two-delay example

For intuition, first consider an multi path fading channel where

Each path has delay either $\tau_0 = 0$ or $\tau_1 = T$

Then the baseband output $Y_B(t)$ is

$$
\begin{aligned}
Y_B(t) &= \sum_i a_{B,i} X_B(t - \tau_i) + \tilde{N}(t) \\
&= \sum_{i:\text{ delay } \tau_0} a_i X_B(t) \; + \sum_{i:\text{ delay } \tau_1} a_i e^{-j2\pi f_c T} X_B(t - T) + \tilde{N}(t)
\end{aligned}
$$

$$
Y_B(mT) = \sum_{i:\text{ delay } \tau_0} a_i X[m] + \sum_{i:\text{ delay } \tau_1} a_i e^{-j2\pi f_c \tau_1} X[m-1] + \tilde{N}(mT)
$$

We'll denote $Y_B(mT)$ by $Y[m]$, and $\tilde{N}(mT)$ by $N[m]$.

Then for $m = 1, 2, \ldots$

$$
\begin{aligned}
Y[m] &= \sum_{i:\text{ delay } \tau_0} a_i X[m] \; + \sum_{i:\text{ delay } \tau_1} a_i \, e^{-j2\pi f_c \tau_1} X[m-1] \; + \; N[m] \\
&= h_0 X[m] + h_1 X[m-1] + N[m]
\end{aligned}
$$

where

$$
h_\ell = \sum_{i:\text{ delay } \tau_\ell} a_i \, e^{-j2\pi f_c \tau_\ell} \qquad \text{for } \ell = 0, 1
$$

- Unlike the simple AWGN channel, a fading channel may have *Inter-Symbol Interference* (ISI) due to multiple paths
- The amount of ISI depends on the fading coefficients. E.g.
  - If $|h_1| \geq |h_0|$, then significant ISI
  - If $|h_1| \ll |h_0|$, then ISI is not a problem

Mutipath Resolution

In general, if we have paths with time-invariant delays which are close to one of K values

$$\tau_0 = 0,\ \tau_1 = T,\ \tau_2 = 2T, \quad \ldots, \tau_K = KT$$

then

$$\begin{aligned} Y[m] &= h_0 X[m] + h_1 X[m-1] + \ldots + h_K X[m-K] + N[m] \\ &= \sum_{\ell=0}^{K} h_\ell\, X[m-\ell] + N[m] \end{aligned}$$

where

$$h_\ell = \sum_{i:\ \text{delay}\ \approx \tau_\ell} a_i\, e^{-j2\pi f_c \tau_\ell} \qquad \text{for } \ell = 0, 1, \ldots, K$$

- h_ℓ is called the ℓth complex *filter tap*
- h_ℓ is the sum over all paths that have delay in $[\ell T - \frac{T}{2}, \ell T + \frac{T}{2}]$
- $X[.]$ filtered with $\{h_\ell\}$; $Y[.]$ is a noisy version of filter output.

$$Y[m] = \sum_{\ell=0}^{K} h_\ell\, X[m-\ell] + N[m]$$

How many taps K does a mutipath fading channel have?

Delay spread T_d is the maximum difference between path delays:

$$T_d = \max_{\text{all paths } i,j} |\tau_i - \tau_j|$$

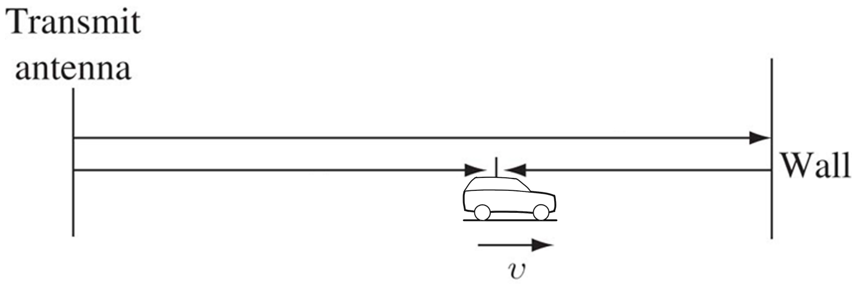
where τ_i denotes the delay of path i

Recall that symbol time $T \approx \frac{1}{2W}$, W is the baseband bandwidth.

- If $T_d \ll T \approx \frac{1}{2W}$, channel has only a single channel tap The channel is said to have **flat fading** (no ISI)
- If $T_d > T \approx \frac{1}{2W}$, channel has multiple taps and is said to be **frequency selective**

- The *coherence bandwidth* of the channel is $W_c = \frac{1}{2T_d}$
- For bandwidths $W \ll W_c$ the channel is flat, for $W > W_c$ it frequency-selective and has $\approx W/W_c$ taps

Time Variations



So far we assumed that the channel was time invariant

But the propagation delays and attenuations of the paths change as the mobile moves

Time-varying discrete channel model:

$$Y[m] = \sum_{\ell} h_\ell[m]\, X[m-\ell] + N[m]$$

Doppler Spread

$$Y[m] = \sum_{\ell} h_\ell[m]\, X[m-\ell] + N[m]$$

The ℓth channel filter tap is now

$$h_\ell[m] = \sum_{i:\tau_i(t) \text{ near } \ell T} a_i(t) e^{-j2\pi f_c \tau_i(t)}, \quad \text{at } t = mT$$

- Paths i whose delay is closest to ℓT contribute to tap ℓ
- If τ_i varies by $\pm\frac{1}{4f_c}$, causes significant change in h_ℓ. Why?

The *Doppler shift* of ith path $= f_c \tau_i'(t)$

The *Doppler spread* $D_s = \max_{i,j} |f_c\tau_i'(t) - f_c\tau_j'(t)|$

The *Coherence time* $T_c = \frac{1}{D_s}$

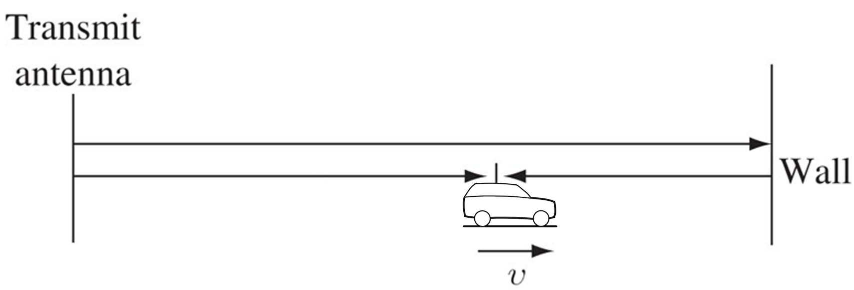
$T_c \approx$ the time over which $h_\ell[m]$ changes significantly

Typical Values for Channel Parameters

Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$D = f_c v/c$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 μs
Coherence bandwidth	$W_c = 1/(2T_d)$	500 kHz

From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

Underspread Channels



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- Coherence time T_c depends on carrier frequency f_c and vehicular speed, of the order of milliseconds
- Delay Spread T_d depends on distance to scatterers, of the order of nanoseconds (indoor) or microseconds (outdoor)
- Hence $T_d \ll T_c$ (usually) and such a channel is said to be *underspread*

Types of Channels

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Underspread	$T_d \ll T_c$

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If we can tolerate a delay of N symbols, i.e., $\frac{N}{2W}$ seconds, then:

- If $T_c \ll \frac{N}{2W}$, then channel changes significantly over the duration of N symbols (Fast Fading)
- We can transmit coded symbols over many "independent" channel realisations

Statistical Models of Fading

$$Y[m] = \sum_{\ell} h_{\ell}[m]\, X[m-\ell] + N[m]$$

- The channel filter taps $h_{\ell}[m]$ need to be measured – typically done using pilot symbols
- But probabilistic models for the fading channel are important to get insight into designing and analysing wireless systems

Noise is AWGN

$$N[m] = N^r[m] + j\, N^i[m]$$

where $N^r[m]$ and $N^i[m]$ are i.i.d. $\sim \mathcal{N}(0, \frac{N_0}{2})$ for all m

Next, we have to model the filter taps $h_{\ell}[m]$ ...

Recall that

$$h_\ell = \sum_{i:\tau_i \text{ near } \ell T} a_i \, e^{-j2\pi f_c \tau_i}$$

Rayleigh fading model:

- h_ℓ is the sum of contributions from many small scattered paths with delay near ℓT.
- We model this as a *complex* Gaussian random variable, with $\text{Re}(h_\ell) \sim \mathcal{N}(0, \frac{\sigma_\ell^2}{2})$, and $\text{Im}(h_\ell) \sim \mathcal{N}(0, \frac{\sigma_\ell^2}{2})$
- The real and imaginary parts are assumed to be independent.

The squared magnitude $|h_\ell|^2$ is then *exponentially* distributed:

$$f_{|h_\ell|^2}(x) = \frac{1}{\sigma_\ell^2} \exp\left(\frac{-x}{\sigma_\ell^2}\right), \quad x \geq 0$$

Notation

$\mathcal{CN}(0, \sigma^2)$ denotes a *complex* Gaussian random variable with

- Its real and imaginary parts are i.i.d, and
- They each have distribution $\mathcal{N}(0, \frac{\sigma^2}{2})$

Thus $N[m] \sim \mathcal{CN}(0, N_0)$ and $h_\ell[m] \sim \mathcal{CN}(0, \sigma_\ell^2)$