4F5: Advanced Wireless Communications

Handout 6: Bit-Interleaved Coded Modulation

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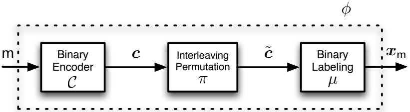
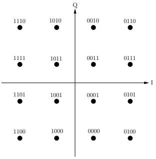
Outline

Basic Scheme

Basic Scheme

- So far we have studied binary codes
- Can we also construct good codes over M-ary signal sets like QAM/PSK?
- Can we do this using well-known binary codes?
- Bit-interleaved coded modulation (BICM)
  - is a practical scheme that uses binary codes to construct non-binary codes
  - used in most communications standards (DVB-S2, WLAN, WiMax, DSL...)
  - de-facto standard for coding over M-ary signal sets like QAM/PSK
  - binary code + interleaver + binary labeling $\mu : \mathbb{F}_2^m \to \mathcal{X}$, such that $\mu(c_1, \dots, c_m) = x$





Decoding

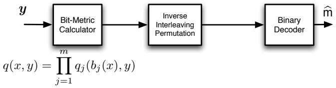
Decoding

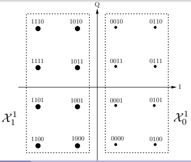
- Due to the interleaver, maximum likelihood only possible by exhaustive search
- We use a maximum metric decoder
- We used to produce symbol metrics $q(x, y) = P_{Y|X}(y|x)$
- We now need to produce bit metrics for the binary decoder
- Due to the interleaver, the decoder treats each bit as if it were independent

$$q(x, y) = \prod_{j=1}^{m} q_j(b_j(x), y)$$

  where the function $b_j(x)$ gives the $j - th$ bit of the binary label of x.
- The bit metrics $q_j(b_j(x), y)$ are obtained as the marginal bit probabilities

$$q_j(b_j(x), y) = P_{Y|B_j}(y|b) = \frac{1}{|\mathcal{X}_b^j|} \sum_{x' \in \mathcal{X}_b^j} P_{Y|X}(y|x')$$





BICM Capacity

BICM Capacity

- By following exactly the same steps of the achievability of the channel capacity, we can show that for $\gamma > 0$

$$\bar{P}_e \leq \Pr\left\{ i_q(\boldsymbol{X};\boldsymbol{Y}) \leq \log \frac{|\mathcal{M}|-1}{\gamma} \right\} + \gamma$$

  with

$$i_q(\boldsymbol{x};\boldsymbol{y}) = \log_2 \frac{q(\boldsymbol{x},\boldsymbol{y})}{\sum_{\boldsymbol{x}'} P_{\boldsymbol{X}}(\boldsymbol{x}')q(\boldsymbol{x}',\boldsymbol{y})} = \sum_{i=1}^{n} \log_2 \frac{q(x_i,y_i)}{\sum_{x'} P_X(x')q(x',y_i)}$$

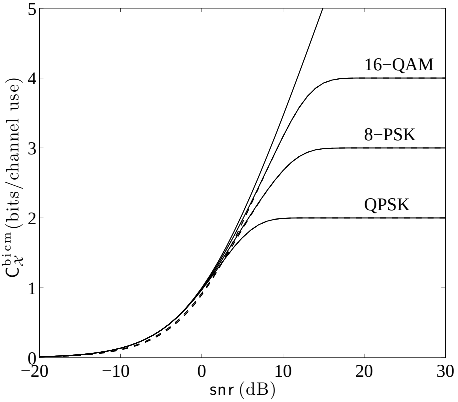
- Therefore, for $n \to \infty$

$$R < \mathbb{E}\left[\log_2 \frac{q(X,Y)}{\sum_{x'} P_X(x')q(x',Y)} \right]$$

  are achievable
- For BICM, $q(x,y) = \prod_{j=1}^{m} q_j(b_j(x),y)$ gives

$$R < \sum_{j=1}^{m} I(B_j;Y)$$

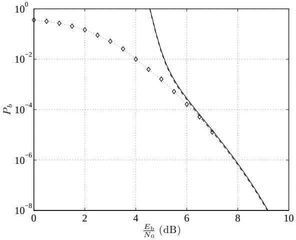
BICM Capacity



Error Probability

Error Probability

- Obtaining closed-form expressions for the PEP is difficult
- We can approximate the PEP as
$$\mathrm{PEP}(\mathbf{0} \to \boldsymbol{c}) \simeq Q\left(\sqrt{2d\,\mathrm{SNR}_{\mathrm{eq}}}\right)$$
where $\mathrm{SNR}_{\mathrm{eq}} = f(\mathrm{SNR}, \mathcal{X}, \mu)$ is an equivalent SNR
- Therefore, BICM preserves the properties of the underlying binary code
  - at large SNR, the term that dominates is that at minimum distance
  - at large SNR $\mathcal{X}$ behaves as a 2-point constellation spaced by $d^{\mathcal{X}}_{\min}$
  - Same behaviour as BPSK, only scaling due to $\mathrm{SNR}_{\mathrm{eq}} < \mathrm{SNR}$



Iterative Decoding

Iterative Decoding

- What if 'somebody' gives us a priori probabilities on the bits?
- We can incorporate this knowledge into the metric
$$q_j(b_j(x), y) = P_{Y|B_j}(y|b) = \sum_{x' \in \mathcal{X}^j_b} P_{Y|X}(y|x') \prod_{j' \neq j} \Pr\{b_{j'}(x')\}$$
- And iterate!