# 4F5: Advanced Wireless Communications

## Handout 7: Characterisation of Fading Channels

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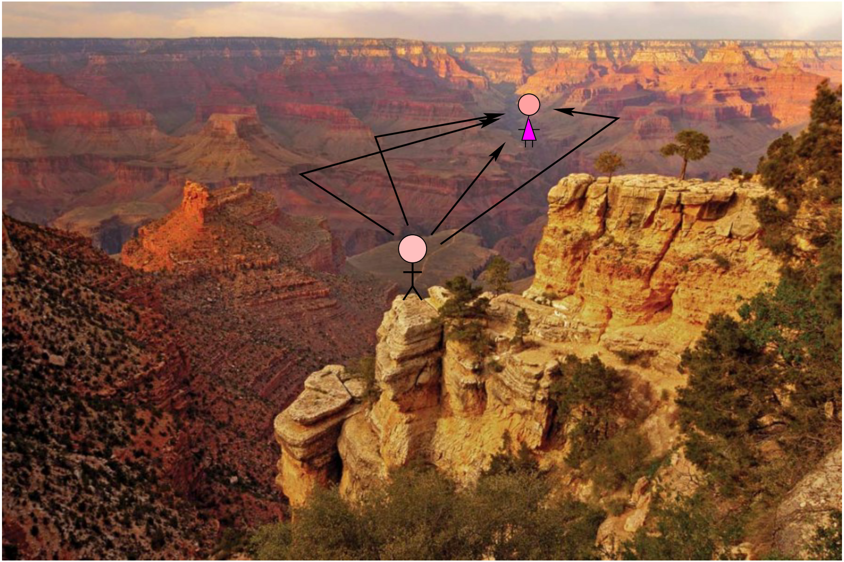
Lent 2012

# Multipath Fading Channels



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- Let $\alpha_n$ denote the attenuation of path $n$, and let $\tau_n$ denote its delay.
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- Let $x_l(\cdot)$ be the baseband equivalent of $x(\cdot)$, i.e., $x(t) = \mathrm{Re}\{x_l(t)e^{j2\pi f_c t}\}$:

$$\begin{aligned} y(t) &= \sum_n \alpha_n(t)x(t-\tau_n(t)) + n(t) \\ &= \mathrm{Re}\left\{\left[\sum_n \alpha_n(t)e^{-j2\pi f_c \tau_n(t)} x_l(t-\tau_n(t)) + n'(t)\right] e^{j2\pi f_c t}\right\} \end{aligned}$$

- Suppose that $\tau_n(t)$ varies only little:

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- Path attenuations and delays are deterministic.
- It is very complicated to describe each path (there are too many!). We therefore model the path attenuations as random processes.

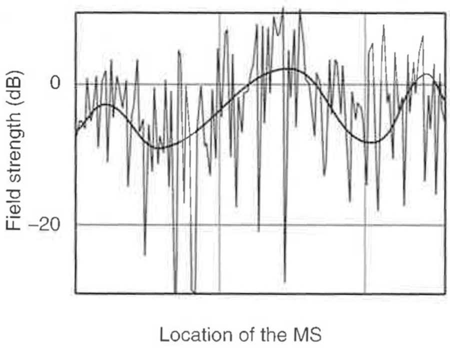


Image from A. F. Molisch, *Wireless Communications*, John Wiley & Sons, 2011.

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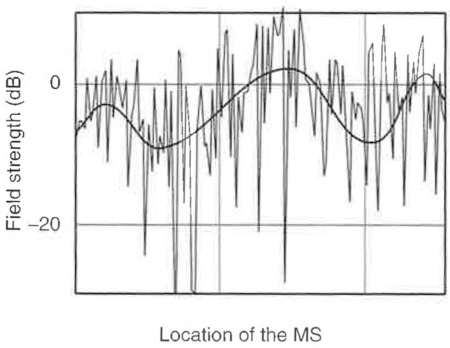


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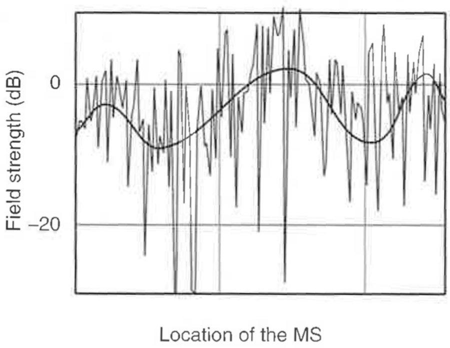


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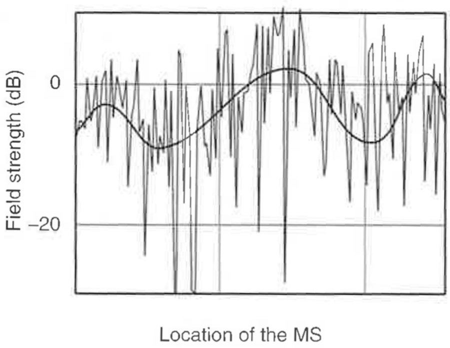


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## Channel Correlation Functions and Power Spectra

- Time-varying impulse response $h(t;\tau)$ is modelled as complex random process.
- Let $R_h(t_1, t_2; \tau_1, \tau_2) \triangleq \mathbb{E}[h(t_1;\tau_1)^* h(t_2;\tau_2)]$. The WSSUS assumption is:
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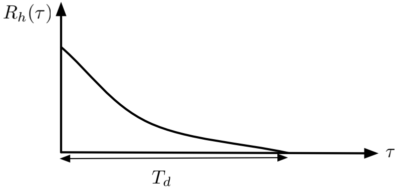

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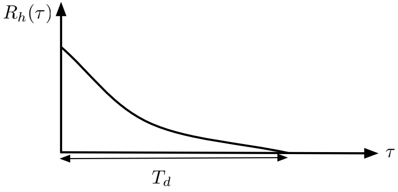

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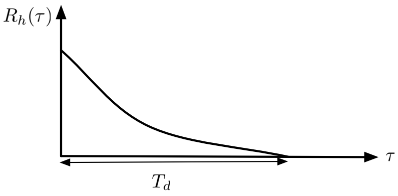

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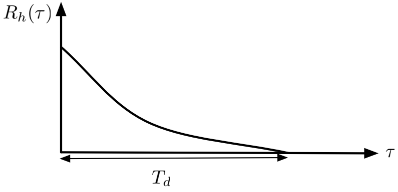

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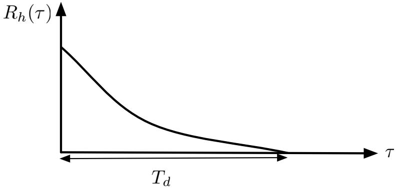

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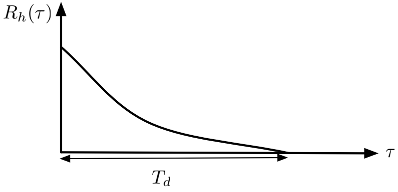

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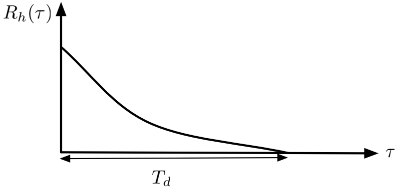
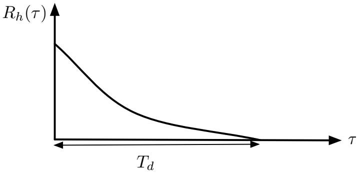


## Channel Correlation Functions and Power Spectra

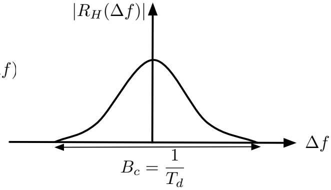
- Let $H(t;f) = \int h(t;\tau)e^{-j2\pi f\tau}d\tau$ be the time-varying Fourier transform of $h(t;\tau)$, and let $R_H(t_1,t_2;f_1,f_2) \triangleq \mathbb{E}[H(t_1;f_1)^* H(t_2;f_2)]$.
- By the WSSUS assumption

$$R_H(t_1,t_2;f_1,f_2) = \int R_h(t_1-t_2,0;\tau,\tau)e^{-j2\pi\tau(f_2-f_1)}d\tau = R_H(t_1-t_2,0;f_1-f_2,0)$$

  Consequently $R_H(\Delta f) \triangleq R_H(0,0;\Delta f,0) = \int R_h(\tau)e^{-j2\pi\Delta f\tau}d\tau$.
- We define the coherence bandwidth as $B_c \triangleq \frac{1}{T_d}$.Two sinusoids whose frequencies are separated by more than $B_c$ are affected independently by the channel.
- Let $B$ denote the bandwidth of the transmitted signal. If $B > B_c$ then the channel is said to be frequency-selective, and if $B \ll B_c$ then it is said to be frequency-flat.




$R_h(\tau) \longleftrightarrow R_H(\Delta f)$



# Multipath Fading Channels

## Channel Correlation Functions and Power Spectra

- Let $H(t; f) = \int h(t; \tau) e^{-j2\pi f \tau} d\tau$ be the time-varying Fourier transform of $h(t; \tau)$, and let $R_H(t_1, t_2; f_1, f_2) \triangleq \mathbb{E}[H(t_1; f_1)^* H(t_2; f_2)]$.
- By the WSSUS assumption

$$R_H(t_1, t_2; f_1, f_2) = \int R_h(t_1 - t_2, 0; \tau, \tau) e^{-j2\pi\tau(f_2 - f_1)} d\tau = R_H(t_1 - t_2, 0; f_1 - f_2, 0)$$

  Consequently $R_H(\Delta f) \triangleq R_H(0, 0; \Delta f, 0) = \int R_h(\tau) e^{-j2\pi \Delta f \tau} d\tau$.
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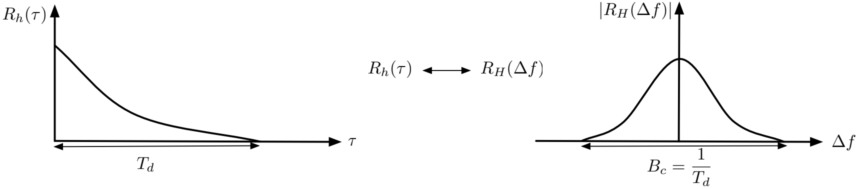

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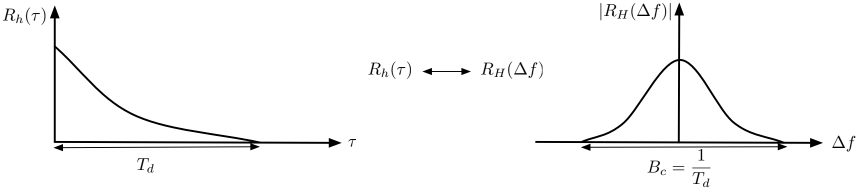

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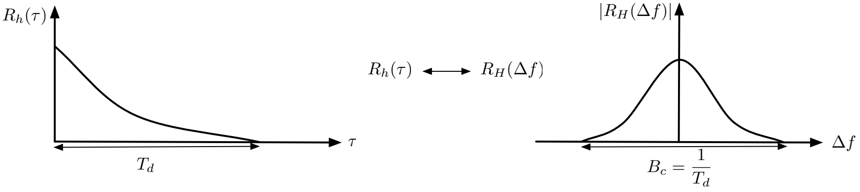

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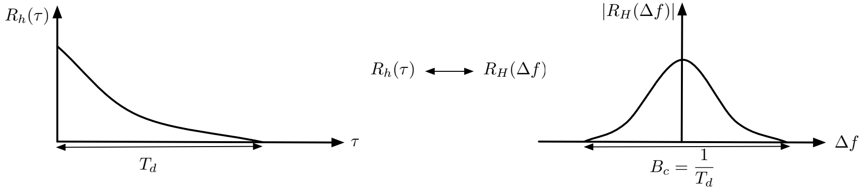

## Channel Correlation Functions and Power Spectra

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$$S_H(\xi;\Delta f) \triangleq \int R_H(\Delta t, 0; \Delta f, 0) e^{-j2\pi\xi\Delta t} d\Delta t \quad \text{and} \quad S_H(\xi) \triangleq S_H(\xi; 0)$$

- $S_H(\xi)$ is called the Doppler power spectrum. It indicates the amount of spectral broadening in the received signal due to Doppler shift.
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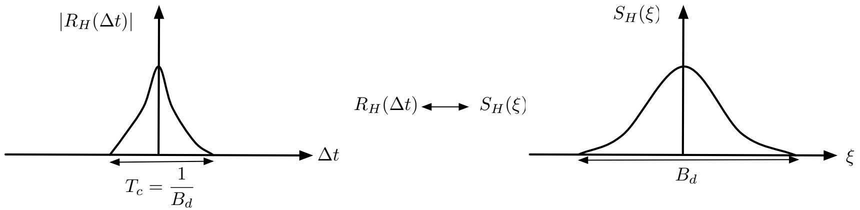

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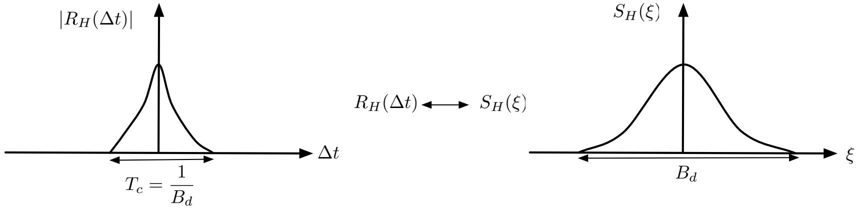

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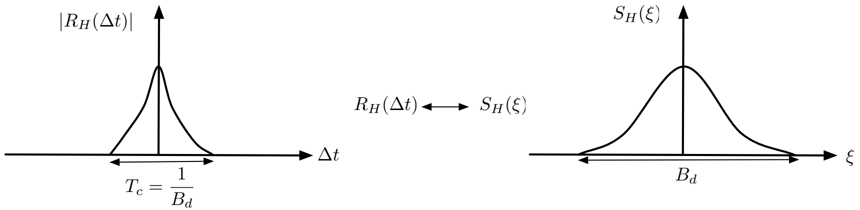

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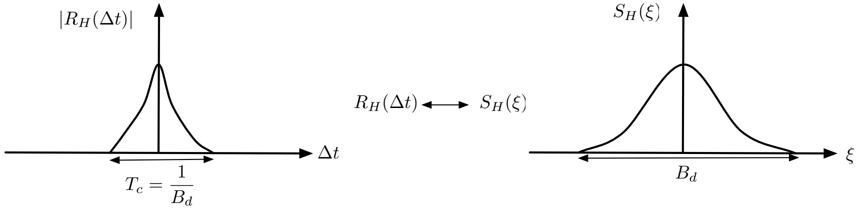

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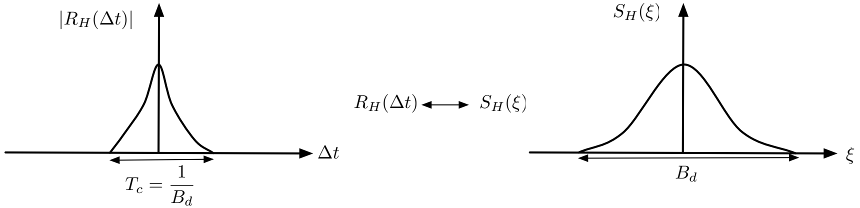

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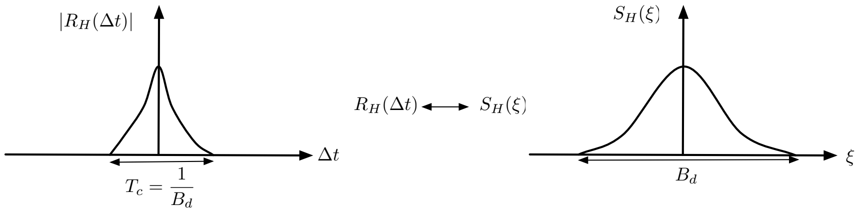

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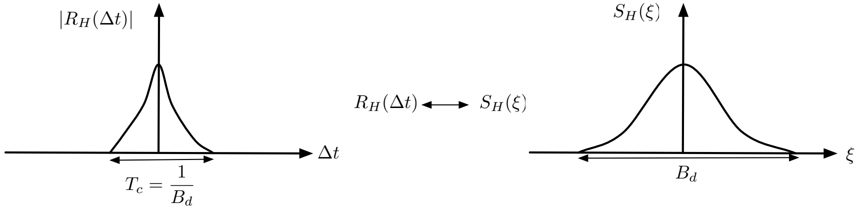

# Multipath Fading Channels

## Example

- Mobile radio channels depend critically on the type of terrain.
- Urban and suburban areas: $T_d \approx 1 - 10\mu$s.
- Rural, hilly areas: $T_d \approx 10 - 30\mu$s.
- A widely used model for the Doppler power spectrum is Jakes' model

$$S_H(\xi) = \begin{cases} \frac{1}{\pi f_m} \frac{1}{\sqrt{1-(\xi/f_m)^2}} & |\xi| < f_m \\ 0 & |\xi| \geq f_m \end{cases}$$

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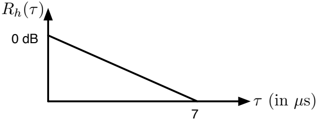
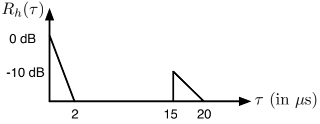
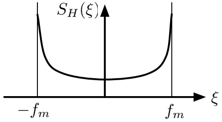



Urban and suburban areas




Hilly terrain



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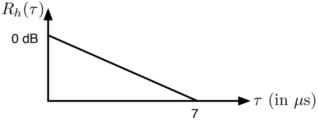


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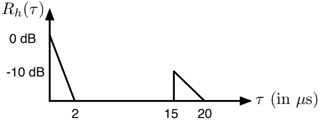


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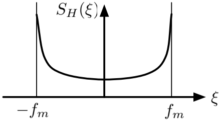

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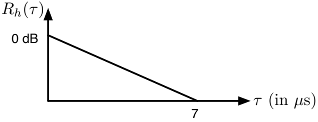


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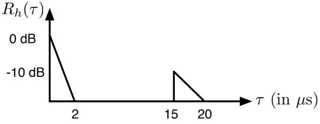


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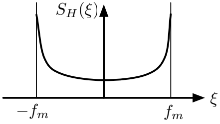

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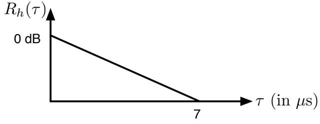


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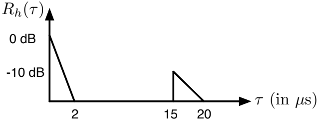


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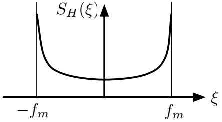

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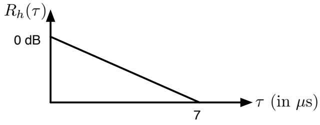


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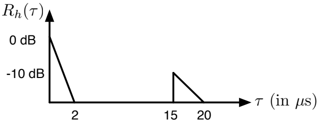


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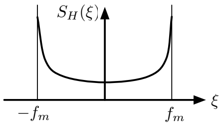
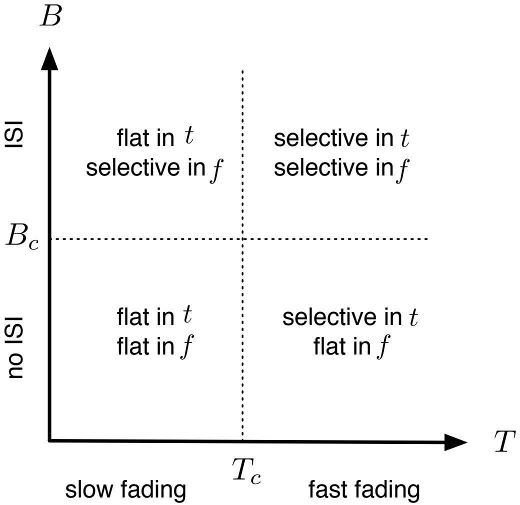


Let $T$ be the codeword duration, and let $B$ be the signal bandwidth:



# Multipath Fading Channels

## Classification of Fading Channels

- Whether channel is selective in time/frequency depends on the statistics of the channel **and** the transmitted signal.
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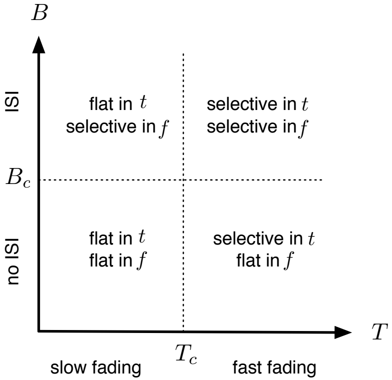

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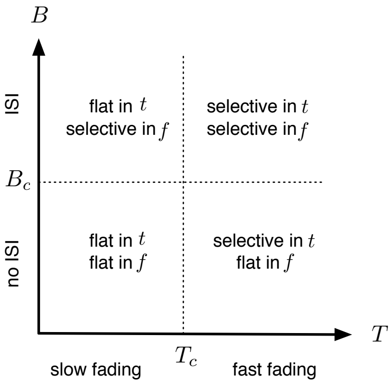

# Multipath Fading Channels

## Classification of Fading Channels

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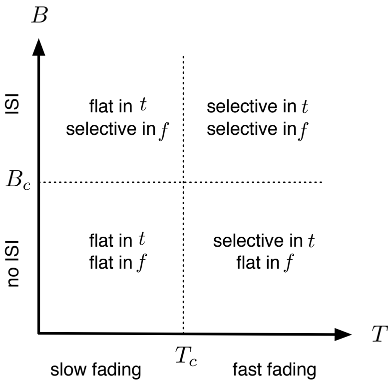


## From Continuous-Time to Discrete-Time

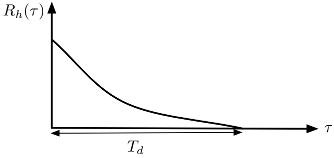
- Continuous-time fading model:

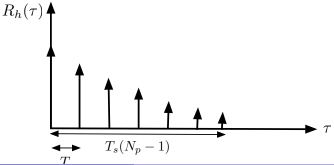
$$y_l(t) = \int h(t;\tau)x_l(t-\tau)\mathrm{d}\tau + n'(t), \qquad t \in \mathbb{R}$$

- Discrete-time fading model:

$$y_i = \sum_{\ell=0}^{N_p-1} h_{i,\ell}x_{i-\ell} + n_i, \qquad i = 1, 2, \ldots$$

- Each path $h_{i,\ell}$ summarises the contribution of multiple scatterers within a symbol period $T_s$.
- Often paths are modelled as complex Gaussian random variables with variance $\sigma_n^2 = |R_h(nT_s)|^2$. This is “justified" by the central limit theorem.
- Other models can be used to better fit experimental data, e.g., Nakagami-$m$.



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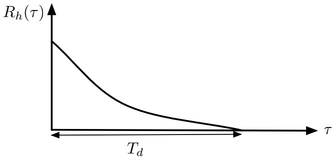


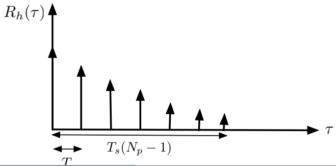

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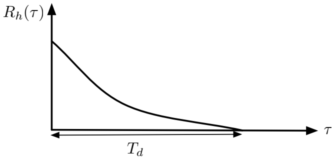


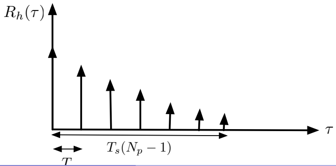

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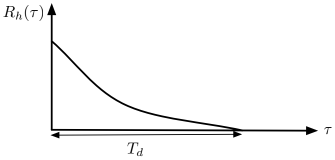


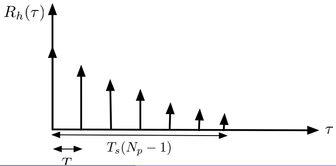

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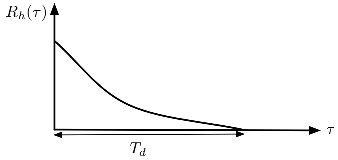


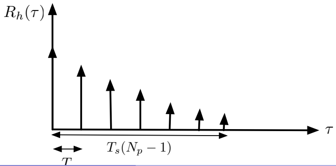

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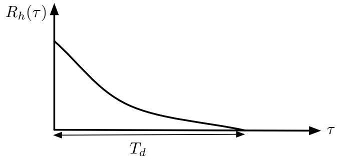


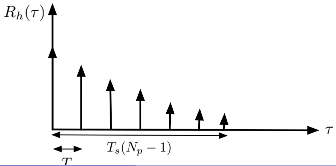

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## Flat Fading

Only one tap, i.e., $N_p = 1$:

$$y_i = h_i x_i + n_i, \qquad i = 1, 2, \ldots$$

- No ISI.
- Slow fading: $h_i$ does not depend on $i$.
- Fast fading: $h_1, h_2, \ldots$ are i.i.d.

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