4F8 Image Processing and Image Coding
Image Processing Examples: 2016

1. Evaluate the Fourier transform of an image $g(u_1, u_2)$ which consists of a white rectangle, of dimensions $(a_1,\, a_2)$, on a black background and centered at $(0, 0)$. What is the spectrum if the rectangle is centered at $(\mu_1,\, \mu_2)$?
Estimate the image bandwidth using any reasonable and convenient measure of bandwidth.

2. If the image in question 1 is to be sampled on a rectangular grid, what sample spacing is required if the maximum amplitude aliasing components are required to be at least 30 dB below the maximum amplitude of the unaliased signal components?

3. Design a separable low-pass FIR filter with bandwidth equal to the bandwidth calculated in question 1.
Show that the 2-D filtering operation may be achieved by applying a 1-D filter in the horizontal direction and another 1-D filter in the vertical direction. Calculate the computational load for separable and non-separable filters of the same size.

4. A particular image has been sampled correctly at the Nyquist rate but it is required to view the sampled image on a display system that has twice the resolution of the sampled image (ie. the image is $N_1 \times N_2$ pixels but the display system can display $2N_1 \times 2N_2$ pixels). Derive a digital filter which will interpolate the image to the required resolution.
HINT: The original continuous image can be recovered conceptually by analogue low-pass filtering the sampled image; the resulting continuous image can then be sampled at the required rate. Considered together these two operations may be represented as a digital filter operating on the original sampled image.
Comment on the likely computational load and discuss any simpler (but perhaps less ideal) methods that might be considered for interpolation.

5. Show that the 2d cosine window, given by $w(u_1, u_2) = w_1(u_1)w_2(u_2)$ where

$$w_i(u_i) = \begin{cases} \cos(\pi \frac{u_i}{U_i}) & \text{if } |u_i| < U_i \\ 0 & \text{otherwise} \end{cases}$$

has a spectrum given by

$$W(\omega_1, \omega_2) = U_1 U_2 \{\mathrm{sinc}(\pi - \omega_1 U_1) + \mathrm{sinc}(\pi + \omega_1 U_1)\}\{\mathrm{sinc}(\pi - \omega_2 U_2) + \mathrm{sinc}(\pi + \omega_2 U_2)\}$$

Sketch this spectrum using Matlab and comment on why w is not ideal as a window function.

In order to create a better window function consider a $w(u_1, u_2) = w_1(u_1)w_2(u_2)$ where

$$w_i(u_i) = \begin{cases} \alpha + \beta \cos(\pi \frac{u_i}{U_i}) & \text{if } |u_i| < U_i \\ 0 & \text{otherwise} \end{cases}$$

where we assume that $\alpha + \beta = 1$ for $i = 1, 2$. Calculate the spectrum of this new 2d window function. For a range of α use Matlab to plot this spectrum (1d form will do) and hence ascertain what good values of α and β might be, giving reasons.

6. Download the image *moireB.jpg* from the website `http://www-sigproc.eng.cam.ac.uk/Main/4F8_2012` and read this into Matlab (`imread`). Make sure you can view the image in Matlab (eg `imshow`). This image is a 756×622 colour image – to make our lives easier we can work with just one channel, say the first.

a Call this first channel image A. Downsample A by a factor of 2 in both dimensions and redisplay the resulting image – you should be able to see aliasing artefacts in the form of moire fringes.

b Now FFT image A and display its spectrum (with DC level at the centre of the image). How many main frequencies does this image have? Display the spectrum of the downsampled image and try to identify evidence of aliasing in this spectrum.

c Using the resolution of the image and the approximate positions of the highest frequencies visible in the spectrum of A, estimate the minimum resolution we would be able to move to while still avoiding aliasing.

7. To investigate how overlaying images results in interference patterns (Moire fringes), follow the following steps:

a Create a 512×512 pattern of horizontal black and white stripes using Matlab (or any other program) – call this image A. A typical width of stripe could be, say, 8 pixels. Take the central 256×256 part of this image as image B.

b Create a rotated version of image B – rotate by a small angle (say 7 degrees, but you can experiment with other angles). One way of doing this is by rotating the central part of image B using the embedding in image A, although any means of doing this is OK.

c Add the two images together to form image C. Note the interference (if angles are small) patterns in image C.

d Fourier transform images A, B and C and observe their spectra. Explain the interference pattern observed in C via the spectrum of C.

NB In doing this question there is a fair amount of scaling that will need to be carried out on the images in order to observe the frequencies in the Fourier domain (as the dynamic range is large but we are plotting on a 0 to 255 scale).

8. By considering a transformation of coordinates, evaluate the fourier transform of an image $g(u1, u2)$ which consists of a single white diamond with vertices at $(\pm a1, 0)$ and $(0, \pm a2)$, as shown in figure 1 (assume white is assigned a constant value, say A, and the surrounding region is black which is assigned zero).

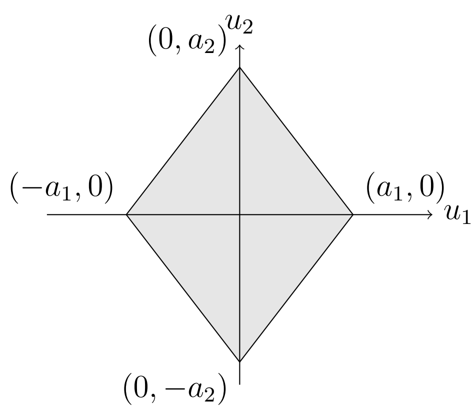

Figure 1:

Estimate the bandwidths of the image stating carefully the measure and axes used.

<u>Solutions</u>

1.

$$G(\omega_1, \omega_2) = A\, a_1 \,\mathrm{sinc}(\omega_1 \frac{a_1}{2})\, a_2 \,\mathrm{sinc}(\omega_2 \frac{a_2}{2})$$

$$G'(\omega_1, \omega_2) = e^{-j(\omega_1\mu_1 + \omega_2\mu_2)} G(\omega_1, \omega_2)$$

Bandwidth measured to first zero in ω_1 and ω_2 directions gives:

$$\text{Bandwidth} = \frac{2\pi}{a_1}, \ \frac{2\pi}{a_2}$$

2.

$$\Delta_1 \leq \frac{a_1}{21} \qquad \Delta_2 \leq \frac{a_2}{21}$$

3. For filter with support $(2M_1 + 1) \times (2M_2 + 1)$ and an image of size $N_1 \times N_2$, the computation requirements are:

Non-separable filter:

$$N_1 \, N_2 \, (2M_1 + 1)(2M_2 + 1)$$

Separable filter:

$$N_1 \, N_2 \, [(2M_1 + 1) + (2M_2 + 1)]$$

4.

$$g(p_1 \frac{\Delta_1}{2}, p_2 \frac{\Delta_2}{2}) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} g(n_1\Delta_1, n_2\Delta_2) \, h[(p_1 - 2n_1)\frac{\Delta_1}{2}, (p_2 - 2n_2)\frac{\Delta_2}{2}]$$

where Δ_1 and Δ_2 are the original sample intervals.

8.

$$G(\omega_1, \omega_2) = 2Aa_1a_2 \text{sinc}\left(\frac{a_1\omega_1 + a_2\omega_2}{2}\right) \text{sinc}\left(\frac{a_1\omega_1 - a_2\omega_2}{2}\right)$$

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