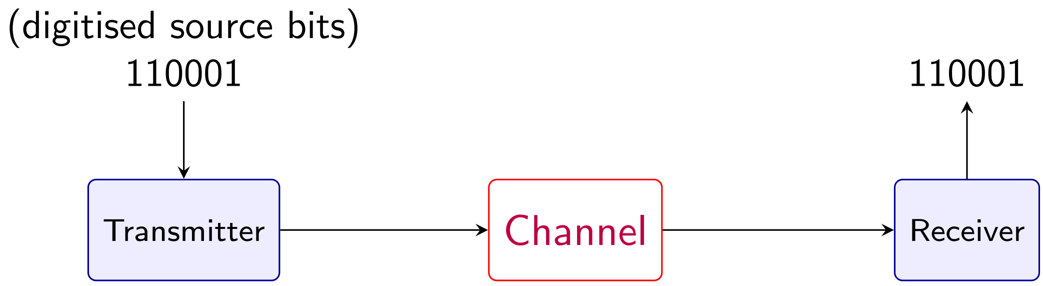
1B Paper 6: Communications

Handout 7: Multiple Access, Course Summary

Single-User Communication



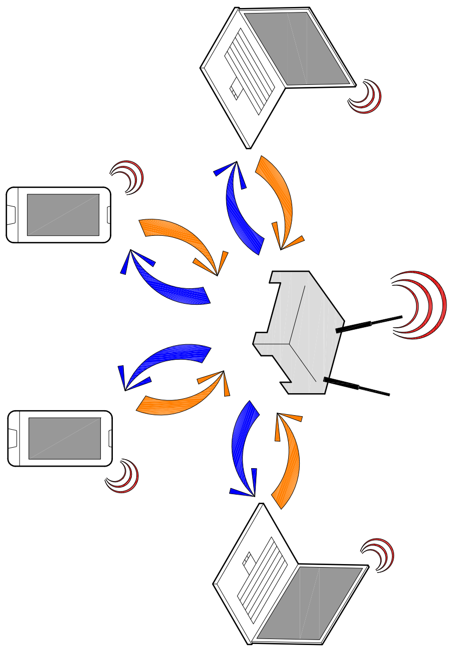
So far we have studied techniques for single-user communication. Recall that:

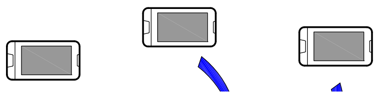
- Transmitter does encoding & modulation
- Receiver does demodulation & decoding
- The user is allocated a channel of bandwidth B

What if there are many users needing to communicate to the receiver using the *same channel bandwidth*?

- How do they share the channel?
- This is the problem of *multiple access*

Multiple-user communication is a typical scenario in wireless networks, cellular communication





Multiple Access: The Main Ideas

Imagine that five of you (multiple users) each have a question to ask me (receiver). What techniques can we use, such that I understand all questions?

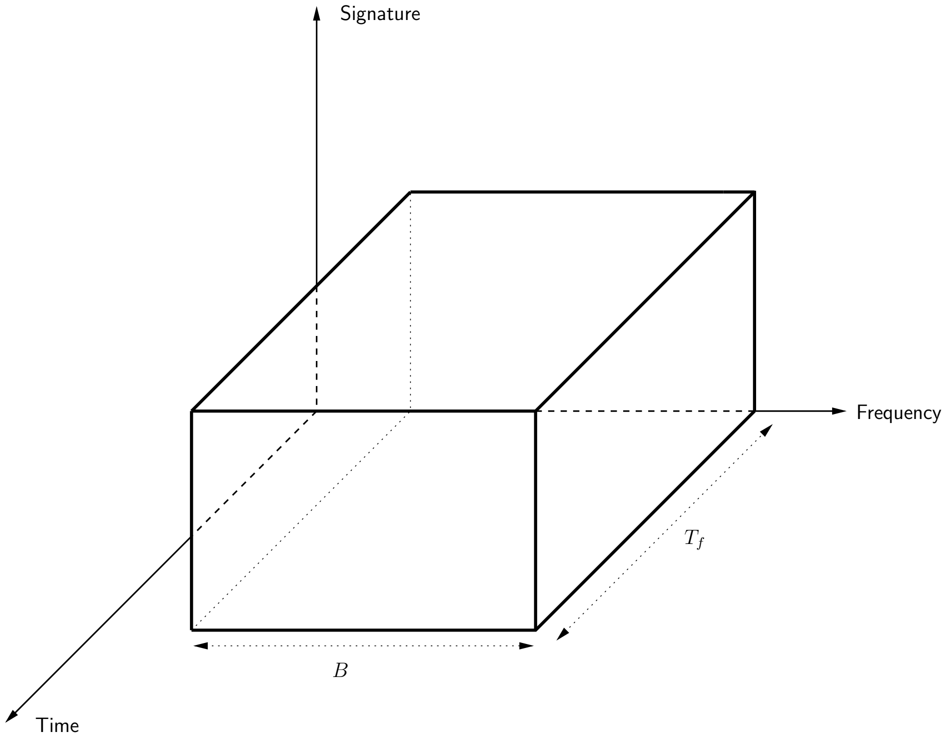
- One after the other, each using the whole bandwidth for a fraction of the time. This is called **time-division** multiple access

- All at the same time, but each with a different frequency. This is called **frequency-division** multiple access. (Each user communicates all the time using using a fraction of the available bandwidth)

- All at the same time using the whole bandwidth, each with a different *signature*, i.e., a different language (known to the receiver). This is called **code-division** multiple access

These three techniques are abbreviated as **TDMA**, **FDMA**, **CDMA**, respectively

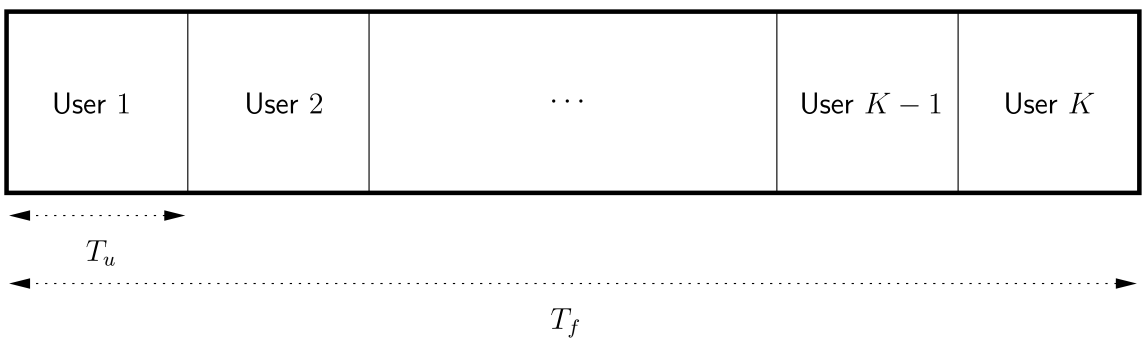
We can think of each multiple-access technique as dividing up a "box" among the users, by cutting along different axes



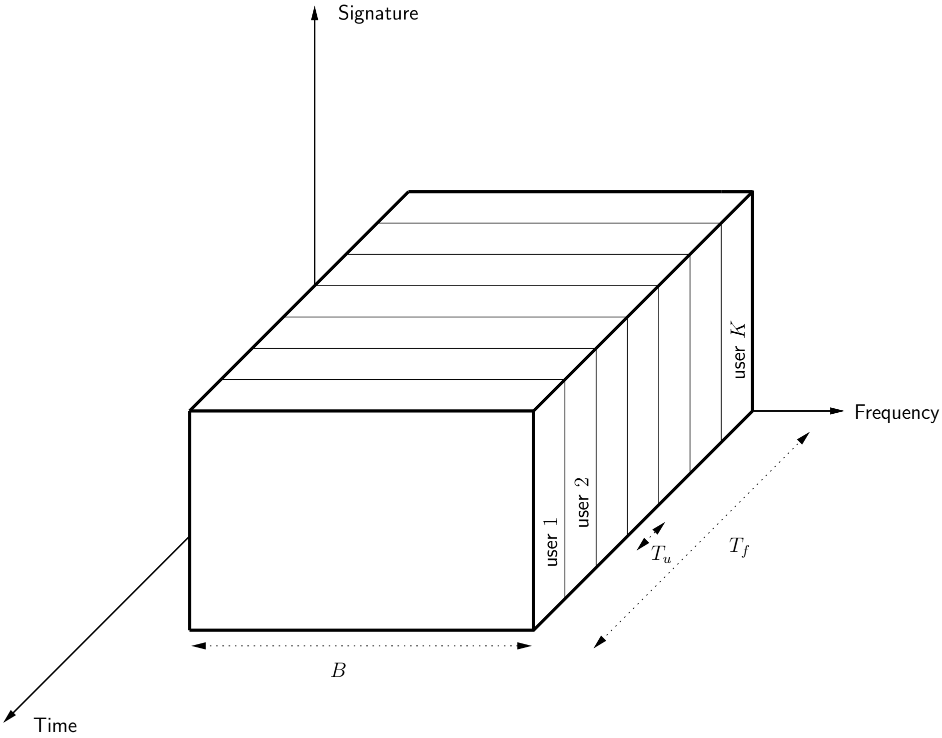
Time Division Multiple Access

In TDMA, multiple users are multiplexed *in time*, so that they transmit one after the other, using the whole bandwidth B.

- Each of K users gets one slot in a *frame* of duration T_f
- K time slots in a frame, each of duration $T_u = \frac{T_f}{K}$



TDMA

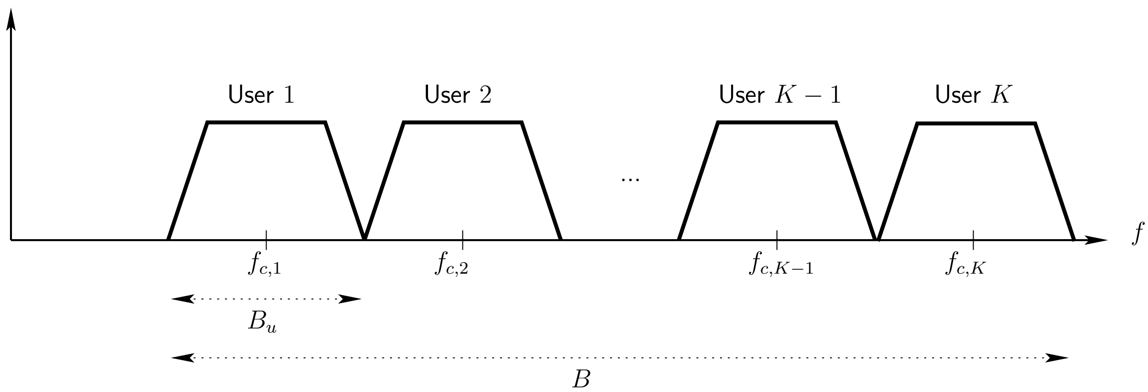


Each user gets $1/K$ *of the box*

GSM, a 2nd generation standard for cellular networks used time-division

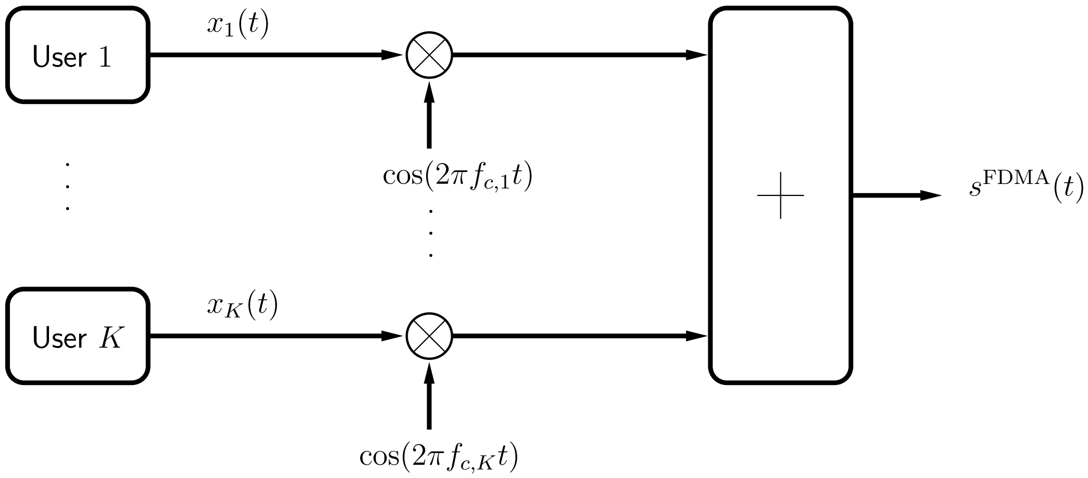
Frequency Division Multiple Access

- In FDMA, K users are multiplexed in the frequency domain by allocating a fraction of the total bandwidth to each one
- They communicate simultaneously on non-overlapping frequency bands of width $B_u < \frac{B}{K}$, so there is no interference

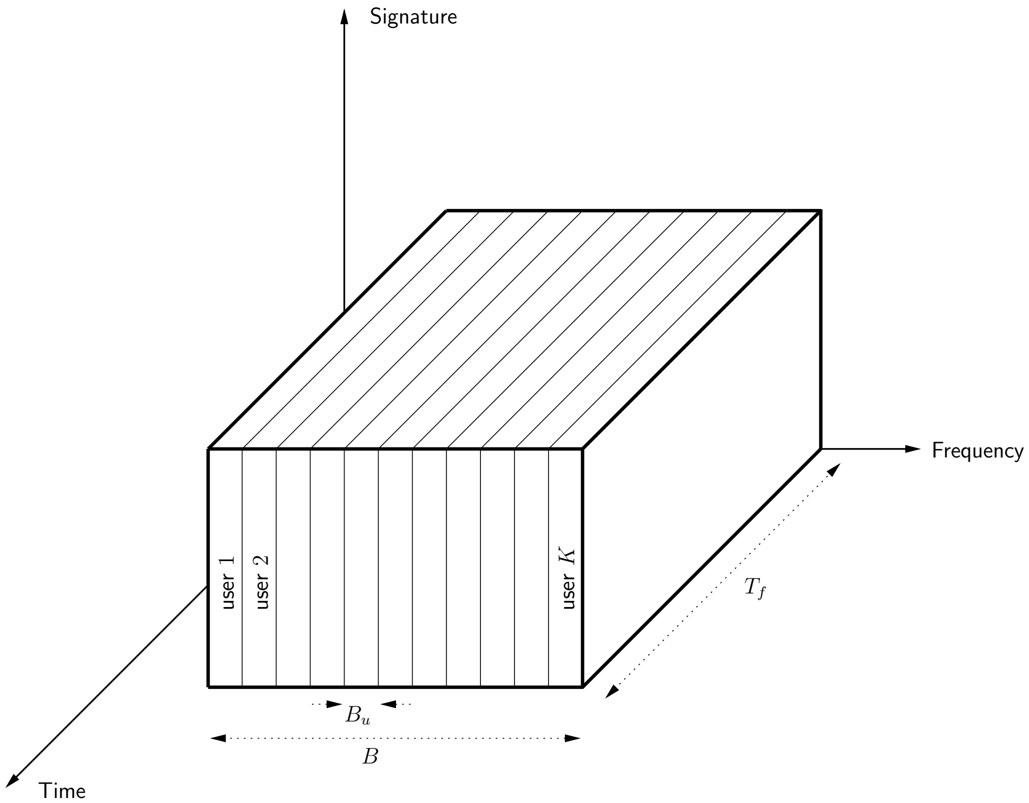


- Can think of each user i having using carrier $f_{c,i}$ to transmit their signal $x_i(t)$, for $i = 1, \ldots, K$

$$s^{\text{FDMA}}(t) = \sum_{i=1}^{K} x_i(t) \cos(2\pi f_{c,i} t)$$



- At the Rx, can separate $x_i(t)$ by multiplying $s^{\text{FDMA}}(t)$ by $\cos(2\pi f_{c,i} t)$ and pass through a filter that is low-pass in the band $\left[-\frac{B_u}{2}, \frac{B_u}{2}\right]$



Each user again gets $1/K$ of the box

A type of FDMA called Orthogonal Frequency Division Multiplexing (OFDM) is used in 4G LTE cellular systems

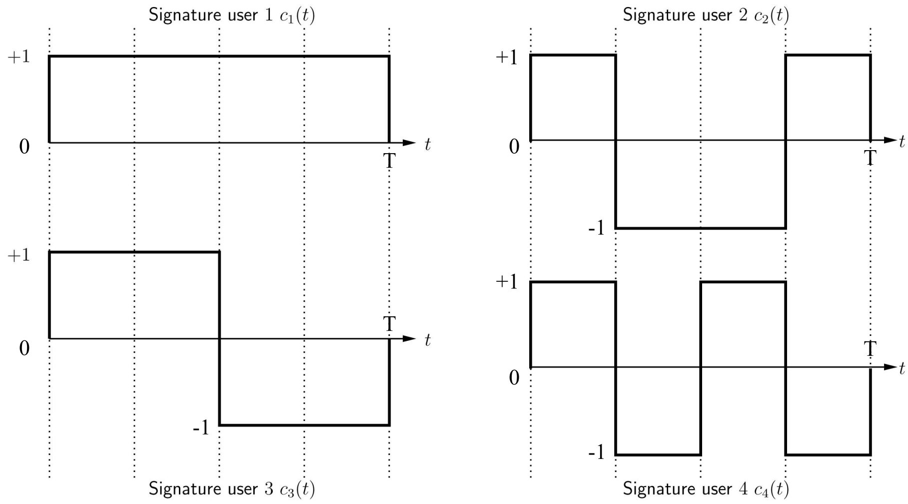
Code Division Multiple Access

- In CDMA, each user is given a unique signature function
- The signatures are denoted $c_i(t)$, $i = 1, \ldots, K$ (K users)

These signatures are chosen to be *orthogonal* over each symbol period T, i.e., for $m = 0, 1, 2, \ldots$

$$\int_{mT}^{(m+1)T} c_i(t)c_j(t)\,dt = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$$

E.g., for $K = 4$ users, the signatures may be



CDMA waveform

- Assume that user i wants to transmit a PAM signal

$$x_i(t) = \sum_k X_k^i p(t - kT)$$

  with a rectangular pulse $p(t)$
- The signals of the K users are multiplexed as

$$s^{\text{CDMA}}(t) = \left[\sum_{i=1}^{K} c_i(t)x_i(t)\right] \cos(2\pi f_c t)$$

- Thus each user i transmits her baseband signal $x_i(t)$ using the entire bandwidth B over the entire time frame of duration T_f
- At the Rx, after down-converting (using product modulator + low-pass filter), we get

$$y(t) = \sum_{i=1}^{K} c_i(t)x_i(t) + \text{ noise}$$

How to separate the users' signals $x_1(t), \ldots, x_k(t)$ at the receiver ?

CDMA receiver

$$y(t) = \sum_{i=1}^{K} c_i(t)x_i(t) + \text{ noise}$$

- At the Rx, signal $x_j(t)$ can be separated by correlating with its signature $c_j(t)$
- Assuming no noise, multiplying $y(t)$ by $c_j(t)$ and integrating, we get

$$\int \left(\sum_{i=1}^{K} c_i(t)x_i(t) \right) c_j(t)\, dt = \sum_{i=1}^{K} \int x_i(t)c_i(t)c_j(t)\, dt$$

$$= \sum_{i=1}^{K} \sum_{m} \int_{mT}^{(m+1)T} x_i(mT)\, c_i(t)c_j(t)\, dt = x_j(t)$$

where we have used (a) $x_i(t)$ is constant over each symbol period, and (b) the orthogonality property of the $c_i(t)$'s
- When the number of users K is large, may only be able to have approximately orthogonal signatures
- All 3G cellular standards use variants of CDMA

The Big Picture

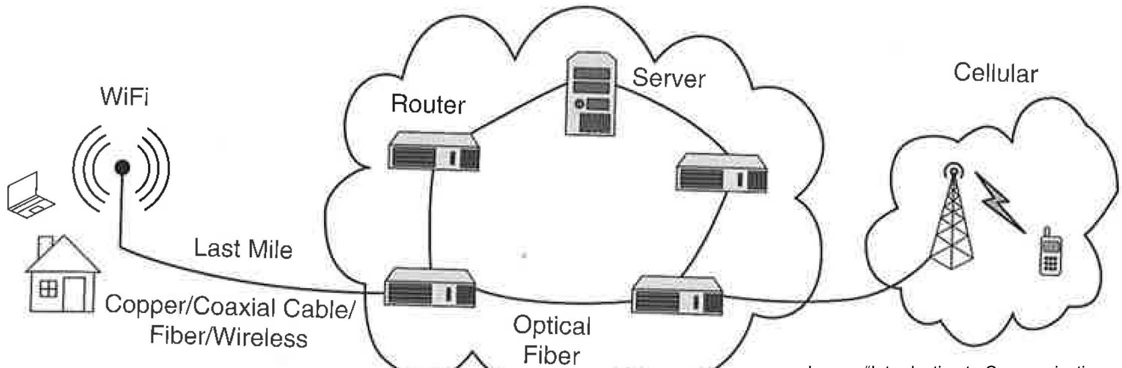
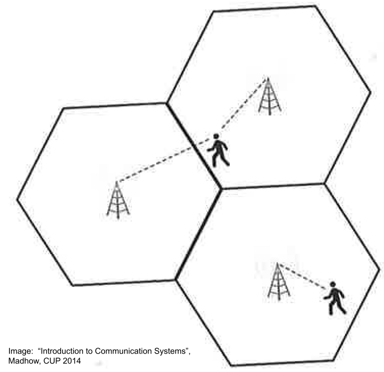


Image: "Introduction to Communication Systems", Madhow, CUP 2014

- The core of the internet consists of routers and servers (data-centres) connected by high-speed optical fibre links
- At the edge, computers are connected by copper wires (DSL) or fibre, and wireless mobile devices connected to wi-fi or cellular (e.g., 3G/4G) networks
- The digital communication design principles we studied in the course apply to each *point-to-point* link of the big network (regardless of the kind of channel)
- Mutliple-access schemes are relevant for wi-fi and cellular networks

Cellular Networks



The network is divided into cells; roughly speaking, there is one base station per cell.

- Each user communicates with the base station in their cell; the base stations are connected to the internet & phone network via high-speed links.
- When a user moves from one cell to another, there is *hand-off*
- Multiple-access schemes such as FDMA or CDMA are needed for users to simultaneously communicate with their base-station; e.g., adjacent cells may use different frequency bands to avoid interference.

Representing and communicating *any* source of information with bits (digitisation) seems routine now . . .

– But was revolutionary in 1948 when Claude Shannon wrote a paper called " *A Mathematical Theory of Communication*"

– The digital revolution of the last few decades has its roots in Shannon's work.

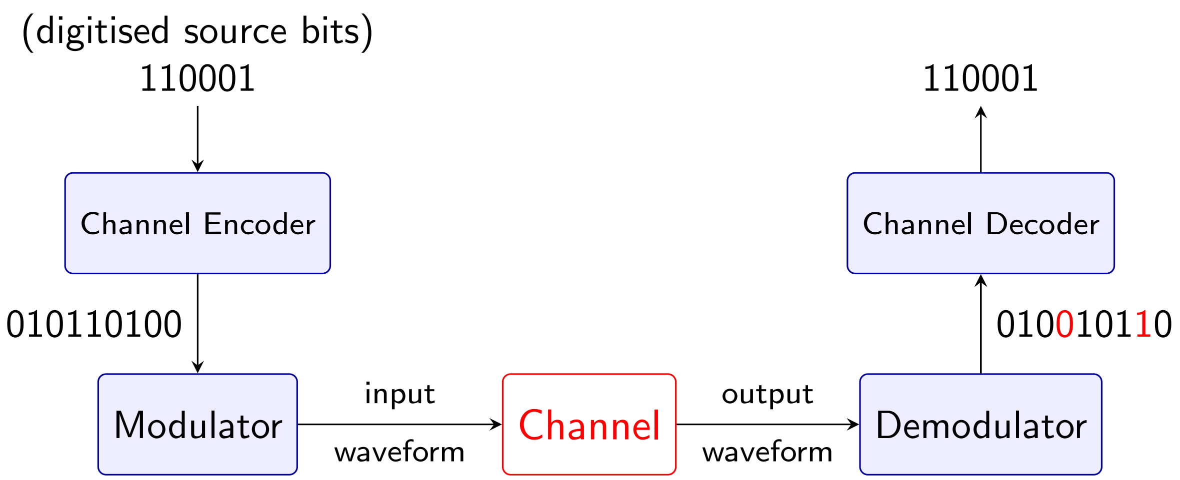
For more on this, you can watch this short documentary on YouTube: *Claude Shannon - Father of the Information Age*

Course Summary

1. *Power, Bandwidth* (Baseband vs Passband) are important resources for communication
2. *Communication channels* can be modelled as linear systems (filters) + noise. If we communicate over a frequency band where the channel frequency response is flat, then we get an additive noise channel.
3. *Analogue Communication*: continuous-time information signal $x(t)$ directly modulates the carrier
   - Variants of Amplitude Modulation: Power, bandwidth, receiver structures
   - FM: Constant power but requires larger bandwidth than AM; Carson's rule for FM bandwidth; more robust to noise
4. *Digitisation*: To convert an analogue source $x(t)$ (e.g., speech/music) to digital:

$$x(t) \quad \xrightarrow{\textit{sampling}} \quad \{x(nT)\}_{n\in\mathbb{Z}} \quad \xrightarrow{\textit{quantisation}} \quad \ldots 0100111 \ldots$$

   Important tradeoff between number of quantiser levels and signal-to-quantisation noise ratio



Digital Communication: Two key parts – Coding and Modulation

5. *Modulation*: Converting bits into a waveform suitable for transmission over the channel
   – PAM for baseband: Tx & Rx structures, bandwidth, power, performance analysis (probability of detection error)
   – QAM for passband: more bandwidth-efficient than PAM
6. *Coding*: Adding redundancy to source bits to make them robust to channel errors
   – An (N, K) block code: K source bits $\longrightarrow$ N code bits $(N > K)$
   – Two simple block codes: $(N, 1)$ repetition code and $(7, 4)$ Hamming code

To conclude, Information Engineering is about:

- **Communication**: Representing information compactly, and transmitting it reliably over noisy channels
- **Signal Processing** : Algorithms to extract clean signals from noisy data (e.g. GPS, medical imaging)
- **Control**: E.g., gyroscope in your smart phone, auto-pilot in an aircraft
- **Statistical Inference & Machine Learning**: Extracting and learning essential features from data to make useful predictions. E.g., Siri in your iPhone

Paper 6 lays the foundation for many of these topics

Relevant Past Tripos Questions (Communications)

From 1B Paper 6:

- 2015, Questions 5(b) and 6
- 2014, Questions 5(b) and 6
- 2013, Question 5
- 2012, Questions 5 and 6
- 2011, Questions 5 and 6, parts (a) and (b)
- 2010, Questions 5 and 6
- 2009, Questions 5 and 6 [note: In 6(c), SNR is defined differently from what we have in Handout 4]
- 2008, Questions 5(e) and 6
- 2007, Questions 4 and 5(a), (b)
- 2006, Question 5 (b),(c)
- 2005, Question 5
- 2004, Question 6 (a), (b), (c)
- 2003, Question 5 excluding the final two lines of part (d)
- 2002, Question 5 excluding part (a)
- 2002, Question 6 except part (c).