Part IB Paper 6: Information Engineering

COMMUNICATIONS

Examples Paper 6/8: Analogue Modulation and Digitisation

1. Show from first principles the following properties of the Fourier transform

   (a) Convolution: $f(t) * g(t) \longleftrightarrow F(\omega)G(\omega)$

   (b) Modulation: $f(t)\cos(\omega_0 t) \longleftrightarrow \frac{1}{2}[F(\omega-\omega_0)+F(\omega+\omega_0)]$

   (c) Parseval's Theorem: $\int |f(t)|^2 dt = \frac{1}{2\pi}\int |F(\omega)|^2 d\omega$

2. Consider a fading channel with 2 paths (direct and reflection) where both transmitter and receiver are static (no movement) and whose input-output relationship in the time domain is given by

$$y(t) = x(t) + \alpha x(t-\tau)$$

   (a) What is the impulse response of this system? Plot the impulse response assuming $\alpha < 1$ and $\tau > 0$. What is the delay spread of the channel? And the coherence bandwidth?

   (b) What is the frequency response? Sketch the magnitude squared of frequency response. Compare with the case with no multipath to explain the effect of the second path in the frequency response.

   (c) Plot the output signal if the input is a rectangular pulse of duration $T \ll \tau$. Sketch the corresponding spectrum. Compare the main-lobe bandwidth with the coherence bandwidth of the channel to explain the output.

3. The Medium Waveband covers frequencies from 500 to 1500kHz. Calculate the number of simultaneous AM transmissions that can be accommodated in this band, if the bandwidth of each modulating signal is 4kHz and a gap of 3kHz is required between sidebands of adjacent signals to prevent crosstalk interference. How many extra transmissions could be accommodated if the modulation format were changed to SSB?

4. An amplitude modulated signal has a waveform defined by

$$s_{\mathrm{AM}} = [10 + a\cos(2\pi f_x t)]\cos(18 \times 10^6 \pi t)$$

   (a) What are the amplitude and frequency (in Hz) of the unmodulated carrier?

   (b) If $a = 3$ and $f_x = 1.5$kHz calculate the modulation index and sketch the waveform.

5. A simple diode-resistor *square-law* circuit can be made with input x V and output y V such that $y = x^2$. The input to the square-law circuit is

$$x(t) = 2 + a_1(t) + a_2(t)$$

   where $a_1(t) = b\cos(2\pi f_x t)$ is a sinusoidal modulating signal and $a_2(t) = \cos(2\pi f_c t)$ is a unit amplitude carrier signal. Show that if the output $y(t)$ is fed into a filter which passes only a suitable frequency band centered on f_c, then the filter output is an AM signal with carrier frequency f_c. What is the modulation index of the AM signal?

6. A frequency modulator converts an input voltage $x(t)$ into an output signal of frequency $f(t)$, such that an input of 1V peak causes a frequency deviation of 50 kHz from the nominal carrier frequency of 10 MHz. Calculate the modulation index and estimate the total bandwidth required for each of the following sine waves:

   (a) 1V peak, frequency 5 kHz

   (b) 1V peak, frequency 10 kHz

   (c) 0.2V peak, frequency 10 kHz

7. Estimate the minimum sampling frequency that can be used for the following signals, if ideal low-pass (anti-aliasing) filters are used:

   (a) A strain gauge generating frequencies up to 100 Hz.

   (b) A telephone speech signal containing frequencies up to 3400 Hz.

   (c) A high quality music signal containing frequencies up to 16 kHz.

   (d) A television signal containing frequencies up to 5.5 MHz.

   Repeat the exercise assuming low-pass (anti-aliasing) filter with a transition bandwidth that is 20% of their pass bandwidth

   Compute the bit rates at the output of a uniform quantiser with 12-bit resolution for (a) and (b), 16 bits for (c) and 8 bits for (d), using the minimum sampling frequencies obtained in the last question.

8. Calculate the rms quantisation noise voltage that would be produced by ideal (i.e. perfectly accurate) analogue-to-digital converters with the following characteristics:

   (a) -5 to $+5$ volt signal range and 8-bit resolution.

   (b) -10 to +10 volt signal range and 12-bit resolution.

   For each case calculate the signal-to-noise power ratio (in dB) that can be achieved if the input signal is (i) a sinusoid covering the full input signal range; (ii) a signal for which the peak value is $2\sqrt{2}$ times the rms value, again scaled to occupy the full input signal range.

   Why would it be inappropriate to estimate quantisation noise voltage in the same way for a square wave signal?

9. (a) An ADC with a -1 to +1 volt signal range and 5-bit resolution is connected to a matching DAC. Given input $x(kT)$ volts, the ADC outputs integer code value m $(-16 \leq m \leq +15)$ such that $m/16$ is the nearest multiple of $1/16$ to $x(kT)$ and the DAC output voltage is then $m/16$V. For example, input sample $x(kT) = 0.1$V gives ADC output code $m = 2$ and output $2/16 = 0.125$V.

   The system is first tested using the signal $x(kT) = 0.9\sin(0.1k\pi)$. [The values of $x(kT)$ for $k = 0, 1, \ldots, 5$ are therefore 0, 0.2781, 0.5290, 0.7281, 0.8560, 0.9000]. It is then tested with a second signal $x_2(kT) = 0.1x(kT)$.

   Compute the actual mean-squared quantisation error which results in each case.

   (b) The same signals are digitised instead of using a companded ADC and matching DAC. These preserve the sign of each input sample $x(kT)$ but cause the *magnitude* of $x(kT)$ to be replaced by the nearest value from the following list:

   0, 0.0280, 0.0561, 0.0841, 0.1122, 0.1346, 0.1615, 0.1938, 0.2326, 0.2791, 0.3349,

   0.4019, 0.4823, 0.5787, 0.6944, 0.8333

   Again, compute the mean-squared quantisation error for the two test signals.

   (c) Re-express the results of (a) and (b) as Signal-to-Noise ratios in dB.

   [A note, for information only: the first five quantisation levels in part (b) are linearly spaced, with spacing 0.028, while the remaining values are in a geometric progression, with each value a factor of 1.2 larger than its predecessor].

Answers:

1.

2.

3. 91, 52

4. a) 10 V, 9 MHz; b) $m_A = 0.3$

5. $m_A = \frac{b}{2}$

6. a) 10, 110 kHz; b) 5, 120 kHz; c) 1, 40 kHz

7. a) 200 Hz; b) 6800 Hz; c) 32 kHz; d) 11 MHz
   a) 240 Hz; b) 8160 Hz; c) 38.4 kHz; d) 13.2 MHz
   a) 2.88 kbit/s; b) 97.92 kbit/s; c) 614.4 kbit/s; d) 105.6 Mbit/s

8. a) 11.28 mV, (i) 49.9 dB, (ii) 43.9 dB; b) 1.41 mV, (i) 74.0 dB., (ii) 68 dB.

   For a square wave signal, the quantising error is not randomly distributed, but is itself a square wave.

9. The exact numerical results will depend on whether you calculated $x(kT) = 0.9\sin(0.1k\pi)$ or used the rounded values given in the question;

   (a) MSE for $x(kT)$: 5.57×10^{-4}: MSE for $x_2(kT)$: 3.77×10^{-4}

   (b) MSE for $x(kT)$: 1.2×10^{-3}: MSE for $x_2(kT)$: 3.15×10^{-5}

   (c) SNRs: linear ADC: 28.6 dB, 10.3dB; companded ADC: 25.2 dB, 21.1 dB.