Communications

IB Paper 6

Handout 2: Analogue Modulation

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Lent Term

Outline

1. Introduction and Motivation
2. Analogue Modulation
   - Amplitude Modulation
   - Phase Modulation
   - Frequency Modulation

Introduction and Motivation

So far...

- We have studied some analog information sources
- We have studied communications channels (attenuation, noise and fading)

Now...

How do we efficiently transmit these signals through the channel?

Remember the following Fourier transform properties for a signal $x(t) \longleftrightarrow X(f)$ and a channel impulse response $h(t) \longleftrightarrow H(f)$:

- $y(t) = h(t) * x(t) \longleftrightarrow Y(f) = H(f)X(f)$
- $s(t) = x(t)\cos(2\pi f_c t) \longleftrightarrow S(f) = \frac{1}{2}[X(f - f_c) + X(f + f_c)]$

Introduction and Motivation

Need for Modulation

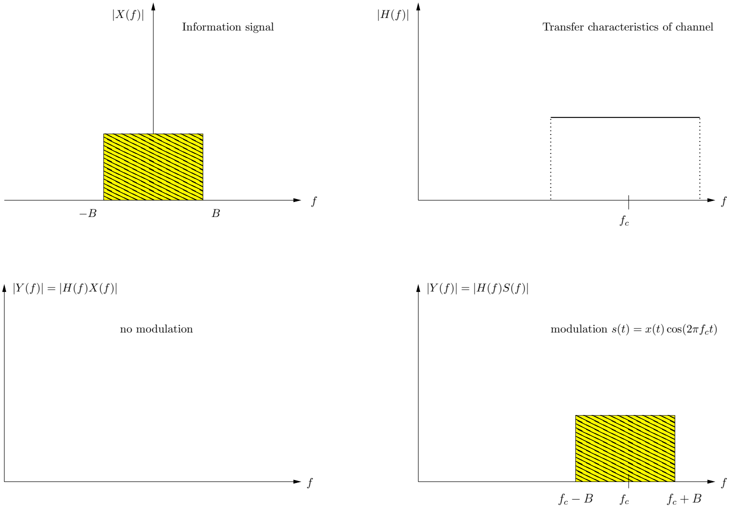
- Communications channels are only able to transmit information (with low attenuation) over certain frequency bands
- In radio transmission, the size of the antennas is usually

$$\frac{\lambda}{2} = \frac{c}{2f}$$

  where λ is the wavelength of the signal and c is the speed of light. For example $f = 300$ Hz, $\frac{\lambda}{2} = 500$ Km!!!
- We need to transmit at frequencies such that we have good propagation characteristics and small antenna size

Introduction and Motivation

A Simple Example: Combining the Convolution and Modulation Properties



Introduction and Motivation

What is modulation?

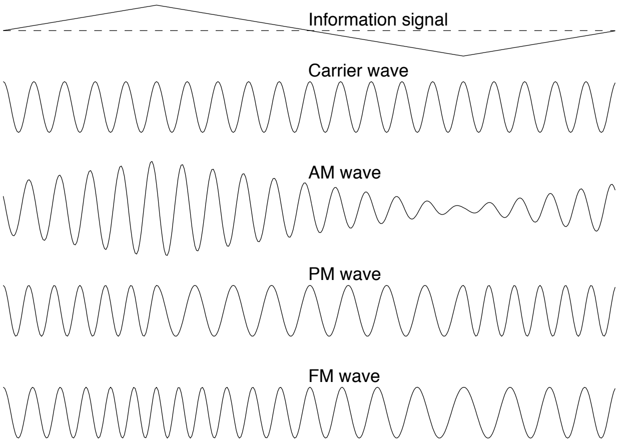
Shaping one or multiple parameters of a carrier wave with the information signal $x(t)$.

$$s(t) = a\cos(2\pi f_c t + \phi)$$

- f_c carrier frequency
- Amplitude Modulation $a = f[x(t)]$
- Angle Modulation
  - Phase Modulation $\phi = f[x(t)]$
  - Frequency Modulation $\phi = 2\pi f[x(t)]t$

Introduction and Motivation

Modulated Signals



Types of Modulation

Depending on the nature of the information signal $x(t)$ we have

- Analogue modulation
- Digital modulation

AM Modulation

- $f[x(t)] = a_0 + x(t)$ so that

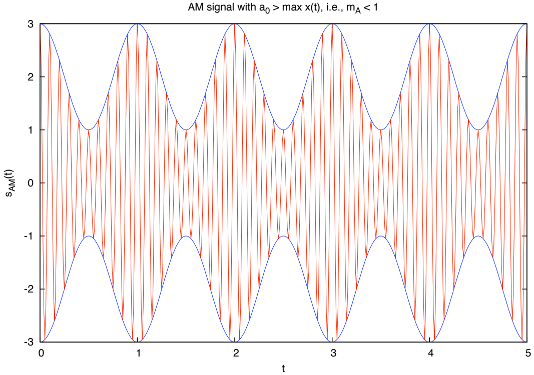
$$s_{\mathrm{AM}}(t) = [a_0 + x(t)]\cos(2\pi f_c t)$$

- We define the modulation index as

$$m_A = \frac{\max_t x(t)}{a_0}$$

the percentage that the carrier's amplitude varies above and below its unmodulated level.

Analogue Modulation
Amplitude Modulation



- $m_A < 1$ desirable, think of extracting the information signal from the modulated signal by envelope detection.
- $m_A > 1$ undesirable, phase reversals would appear, and recovering the information signal is more complex.

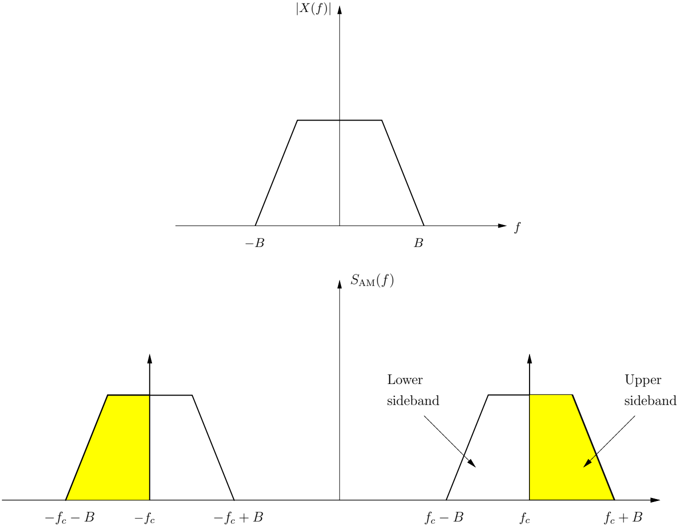
AM Spectrum

We denote the spectrum of $s_{\text{AM}}(t) = [a_0 + x(t)]\cos(2\pi f_c t)$ by $S_{\text{AM}}(f) = \mathcal{F}[s_{\text{AM}}(t)]$ ($\mathcal{F}[.]$ denotes the Fourier transform) and it is given by

$$\begin{aligned} S_{\text{AM}}(f) &= \mathcal{F}[s_{\text{AM}}(t)] \\ &= a_0\mathcal{F}[\cos(2\pi f_c t)] + \mathcal{F}[x(t)] \star \mathcal{F}[\cos(2\pi f_c t)] \\ &= \underbrace{\frac{a_0}{2}[\delta(f - f_c) + \delta(f + f_c)]}_{\text{carrier}} + \underbrace{\frac{1}{2}[X(f - f_c) + X(f + f_c)]}_{\text{information}} \end{aligned}$$

Analogue Modulation

Amplitude Modulation



Properties of AM

1. From the spectrum calculation, we see that the resulting AM modulated signal $s_{\text{AM}}(t)$ occupies a bandwidth
$$B_{\text{AM}} = 2B$$
since both sidebands are transmitted.

2. The transmitted power is
$$P_{\text{AM}} = \frac{a_0^2}{2} + \frac{P_x}{2} = P_c + 2P_{sb}$$
where $P_c = \frac{a_0^2}{2}$ is the carrier power and $P_{sb} = \frac{P_x}{4}$ is the power required to transmit one sideband.

Improving AM: Double Sideband Suppressed Carrier (DSB-SC)

- The carrier transmission wastes power since only a fraction of the total power goes to transmit the information message
- DSB-SC transmits both sidebands but not the carrier: it uses the modulation property of the Fourier transform directly.
- Bandwidth of DSB-SC

  $$B_{\text{DSB-SC}} = 2B$$

  the same as AM
- Power of DSB-SC

  $$P_{\text{DSB-SC}} = \frac{P_x}{2} = 2P_{sb}$$

  which improves on AM since the carrier is not transmitted.

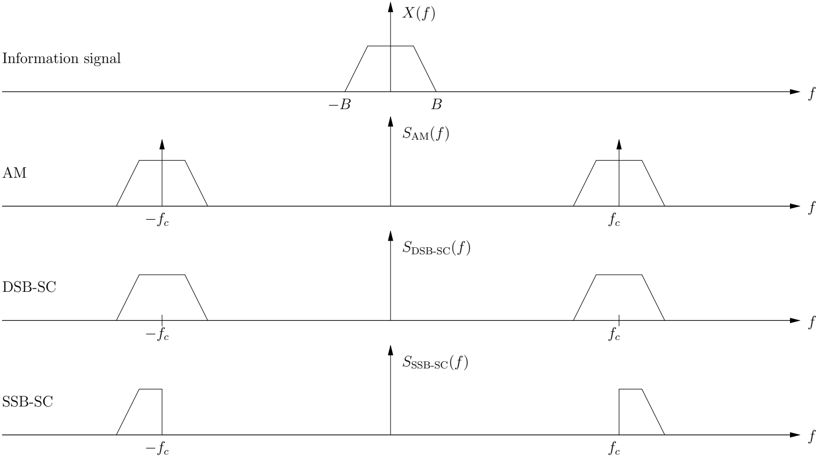
Improving AM: Single Sideband Suppressed Carrier (SSB-SC)

- By symmetry we could obtain one sideband from the other, so transmission of both sidebands is not strictly necessary
- SSB-SC transmits only one sideband and does not transmit the carrier
- Bandwidth of SSB-SC $B_{\text{SSB-SC}} = B$

  half of AM or DSB-SC!
- Power of SSB-SC $P_{\text{SSB-SC}} = P_{sb}$

Analogue Modulation

Amplitude Modulation



Analogue Modulation

Phase Modulation

PM Modulation

We modulate the instantaneous phase of the carrier signal

$$\theta_i(t) = 2\pi f_c t + \phi_\Delta x(t)$$

yielding the PM modulated signal

$$s_{\mathrm{PM}}(t) = a_0 \cos(\theta_i(t)) = a_0 \cos\left(2\pi f_c t + \phi_\Delta x(t)\right)$$

where ϕ_Δ is the *phase deviation* or modulation index of PM.

- Analogue PM is rarely used in practice
- PM has most of the properties of FM
- We will study FM in some detail

FM Modulation

We modulate the instantaneous frequency of the carrier signal

$$f_i(t) = f_c + k_f x(t)$$

which translates into an instantaneous phase equal to

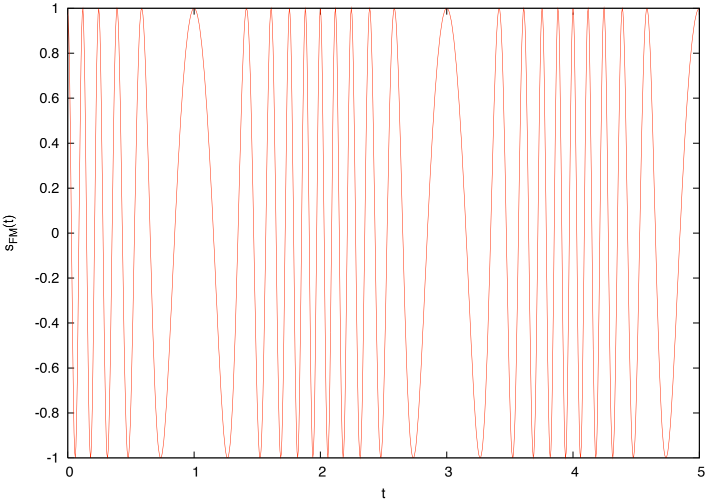
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t x(\tau) d\tau$$

yielding the FM modulated signal

$$s_{\text{FM}}(t) = a_0 \cos(\theta_i(t)) = a_0 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t x(\tau) d\tau\right)$$

Analogue Modulation

Frequency Modulation



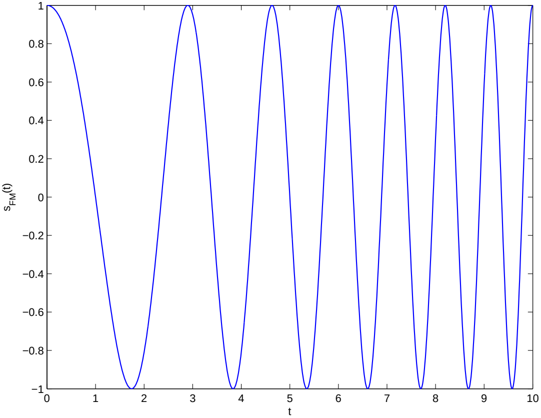
FM Properties

1. Constant transmitted power $P = \frac{a_0^2}{2}$
2. Nonlinearity: $\text{FM}(x_1(t) + x_2(t)) \neq \text{FM}(x_1(t)) + \text{FM}(x_2(t))$
3. FM more robust to noise than AM, since the message is *hidden* in the frequency and not in the amplitude
4. Bandwidth penalty with respect to AM

Analogue Modulation

Frequency Modulation

What information signal does this FM modulated signal correspond to?



(a) a constant, (b) a ramp, (c) a rectangular pulse, (d) no clue

Bandwidth of FM Signals

Consider FM modulation of a tone, $x(t) = a_x \cos(2\pi f_x t)$, then

$$f_i(t) = f_c + k_f a_x \cos(2\pi f_x t)$$

$$\theta_i(t) = 2\pi f_c t + \frac{k_f a_x}{f_x} \sin(2\pi f_x t)$$

We define $\Delta f = k_f a_x$ *frequency deviation* and $m_F = \frac{\Delta f}{f_x}$ *modulation index*, which represents the maximum phase deviation, i.e., the maximum departure of the angle $\theta_i(t)$ from $2\pi f_c t$ of the carrier. Then the FM signal becomes

$$s_{\mathrm{FM}}(t) = a_0 \cos\left(2\pi f_c t + m_F \sin(2\pi f_x t)\right)$$

Analogue Modulation

Frequency Modulation

Bandwidth of FM Signals

Using $\cos(A+B) = \cos A\ \cos B - \sin A\ \sin B$ we can write

$$s_{\mathrm{FM}}(t) = a_0 \cos(2\pi f_c t) \cos\left(m_F \sin(2\pi f_x t)\right)$$
$$- a_0 \sin(2\pi f_c t) \sin\left(m_F \sin(2\pi f_x t)\right)$$

Now using Fourier series we write ($C_n = J_n(m_F)$ are Bessel functions of the first kind)

$$\cos\left(m_F \sin(2\pi f_x t)\right) = C_0 + \sum_{n=1}^{\infty} C_{2n} \cos(4n\pi f_x t)$$

$$\sin\left(m_F \sin(2\pi f_x t)\right) = \sum_{n=1}^{\infty} C_{2n-1} \sin(2(2n-1)\pi f_x t)$$

Bandwidth of FM Signals

Using the relationships $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ and $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$ we obtain

$$\begin{aligned} s_{\mathrm{FM}}(t) = \frac{a_0}{2}\Big\{ & 2\cos(2\pi f_c t) \\ & - C_1\left[\cos(2\pi(f_c - f_x)t) - \cos(2\pi(f_c + f_x)t)\right] \\ & + C_2\left[\cos(2\pi(f_c - 2f_x)t) + \cos(2\pi(f_c + 2f_x)t)\right] \\ & - C_3\left[\cos(2\pi(f_c - 3f_x)t) - \cos(2\pi(f_c + 3f_x)t)\right] \\ & + \dots \Big\} \end{aligned}$$

Bandwidth of FM Signals

Putting everything together and using $J_{-n}(m_F) = (-1)^n J_n(m_F)$

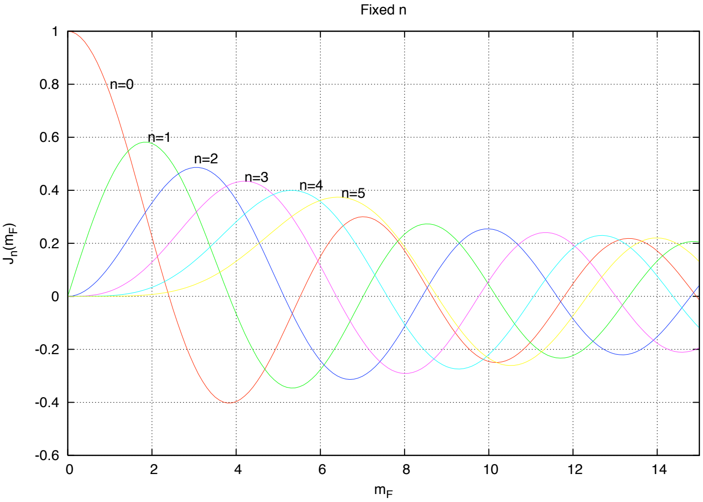
$$s_{\text{FM}}(t) = a_0 \sum_{n=-\infty}^{\infty} J_n(m_F) \cos(2\pi(f_c + nf_x)t)$$

which creates sidebands at harmonics of f_x, i.e., it expands the bandwidth beyond $f_c + f_x$ (that of AM)

$$S_{\text{FM}}(f) = \frac{a_0}{2} \sum_{n=-\infty}^{\infty} J_n(m_F) \left[\delta(f - f_c - nf_x) + \delta(f + f_c + nf_x)\right]$$

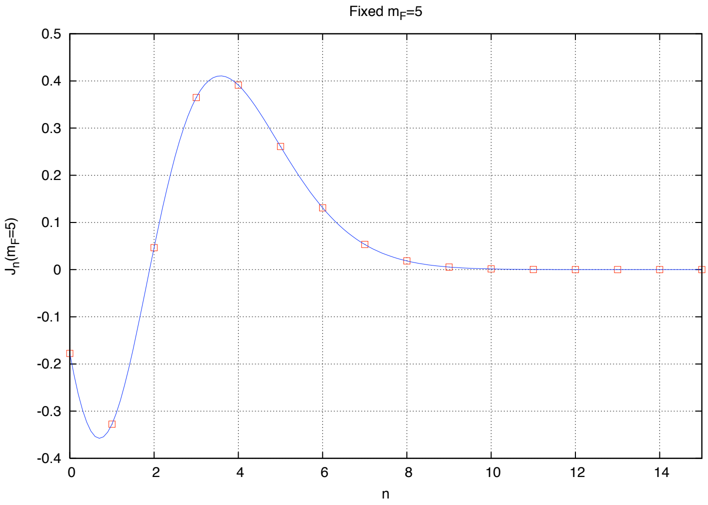
Analogue Modulation

Frequency Modulation



Analogue Modulation

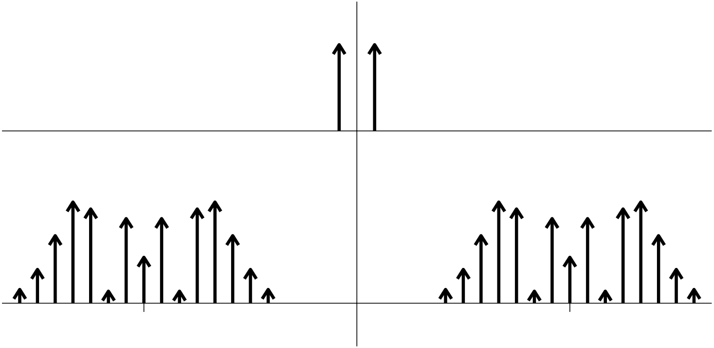
Frequency Modulation



Analogue Modulation

Frequency Modulation

Example: FM spectrum of a pure tone with $m_F = 5$.



Bandwidth of FM Signals: Carson's rule

Carson proposed the following rule to estimate the effective bandwidth of an FM-modulated tone (the absolute bandwidth is infinite, as shown by our calculations in previous slides)

$$B_{\mathrm{FM}} = 2\Delta f + 2f_x = 2\Delta f \left(1 + \frac{1}{m_F}\right)$$

For general signals $x(t)$ of bandwidth B, the *generalised* Carson's rule gives

$$B_{\mathrm{FM}} = 2(\Delta f + B)$$

Example

BBC Radio Cambridgeshire: $f_c = 96$ MHz and $\Delta f = 75$ kHz.
Assuming the voice/music signals have $B = 15$ KHz, we have

$$m_F = \frac{75}{15} = 5$$

and

$$B_{\text{FM}} = 2(\Delta f + B) = 2(75 + 15) = 180kHz,$$

while

$$B_{\text{AM}} = 30kHz$$

Note that FM has better quality (larger SNR – robustness against noise).