3F4 Digital Modulation - Examples Paper 1

1. In the analysis of modulation, certain fundamental relationships between the time and frequency domains are often used. Prove the following relationships from first principles (i.e. assuming only the standard definition of the Fourier transform).

$$\text{If } p(t) \rightleftharpoons P(\omega), \text{ then:} \quad \begin{aligned} p^*(t) &\rightleftharpoons P^*(-\omega) \\ p(t)\ e^{j\omega_C t} &\rightleftharpoons P(\omega - \omega_C) \\ p^*(t)\ e^{-j\omega_C t} &\rightleftharpoons P^*(-(\omega + \omega_C)) \end{aligned}$$

2. Derive an expression for the bit error probability of a binary amplitude shift keyed (ASK) signal, in which the two possible phasors for bit k are 0 and $2g(t - kT_b)$, in terms of the mean energy per bit E_b and the noise density N_0 at the receiver input. (You may assume that the signal pulse $g(t)$ is non-zero only for $0 < t < T_b$, and that the optimum ASK receiver is the same as for BPSK except that the detector threshold is offset from zero by an appropriate amount to minimise the bit error rate with equiprobable bits.)

In terms of E_b/N_0 (in dB), how does this ASK system compare with BPSK ? Can you explain this simply for unshaped ASK and BPSK by regarding ASK as BPSK plus a pure carrier ?

3. Obtain expressions for the power spectral density of a shaped BPSK signal conveying random data when the signal pulses $g(t)$ over the interval 0 to T_b are given by:

a) $g(t) = a_0\ \sin(\pi t/T_b)$
b) $g(t) = a_0[1 - \cos(2\pi t/T_b)]$

You may assume that $g(t)$ is zero outside the above interval.

For each PSD determine the distance from the centre of the main lobe to the first zero and the levels (in dB) of the mid points of the first two side lobes relative to the level at the centre of the main lobe.

How would these power spectra be modified if the modulation were QPSK and the durations of the above pulses were extended to equal the symbol period ?

4. Figure 1 shows the block diagram of a simplified receiver for unshaped QPSK data. The input signal $s(t)$ is given by:

$$s(t) = a_0\ Re[(b_{2k} + j\ b_{2k+1})\ e^{j(\omega_C t + \phi_C)}]$$

What phase shifts ϕ_{ref} of the receiver local oscillator will result in the output $i(t)$ being dependent only on the bit stream b_{2k} and $q(t)$ being dependent only on the bit stream b_{2k+1} ?

The Carrier Loop Phase Detector generates the loop error signal: $e = q\ \text{sgn}(i) - i\ \text{sgn}(q)$

Calculate and sketch the response of this detector to a pure carrier, formed by $b_k = 1$ for all k, as a function of $(\phi_C - \phi_{ref})$ from 0 to 2π (assume phase shifts and attenuation in the dual lowpass filters may be ignored).

If the loop locks at positive-going zero-crossings of the detector, which of the four lock points results in correct demodulation of the data and what effects would locking at each of the other three lock points have on the received data ?

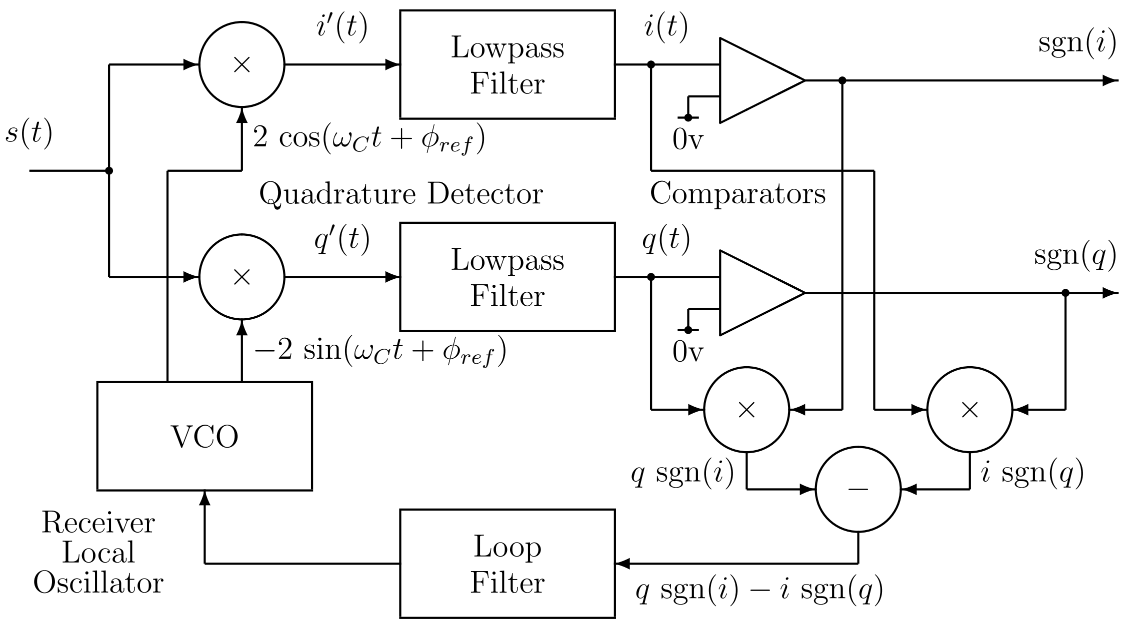

Figure 1: Simplified QPSK Receiver.

5. In the lecture course the bit error performance of a QAM system is calculated based on the assumption of Gray (unit distance) coding for the m bits which contribute to each M-level component of the carrier. For 64-QAM determine the bit error performance if the two 3-bit words are each coded using simple binary instead of Gray coding such that the two outer levels of each component are 000 and 111.

By what factor is the bit error rate increased over that of a Gray coded system, and what increase in signal-to-noise ratio is required in order to compensate for this when the required output bit error rate is 10^{-3} ? (You may use the curves or the approximation formula for $Q(x)$ from the lecture handout for this calculation.)

Relevant past Tripos questions: 3F4 has been running for a number of years and virtually all past Tripos questions are relevant.

Nick Kingsbury, February 11, 2016.

Answers:

2. $P_E = Q(\sqrt{E_b/N_0})$, 3 dB worse than BPSK.

3. a) $\dfrac{a_0^2 T_b}{4}\left[\mathrm{sinc}\dfrac{\omega T_b - \pi}{2} + \mathrm{sinc}\dfrac{\omega T_b + \pi}{2}\right]^2, \; \dfrac{3}{2T_b}$ Hz, -23.52 dB, -30.88 dB;

b) $\dfrac{a_0^2 T_b}{4}\left[2\,\mathrm{sinc}\dfrac{\omega T_b}{2} + \mathrm{sinc}(\dfrac{\omega T_b}{2} - \pi) + \mathrm{sinc}(\dfrac{\omega T_b}{2} + \pi)\right]^2, \; \dfrac{2}{T_b}$ Hz, -32.30 dB, -41.85 dB.

4. $\phi_{ref} = \phi_C + n\pi, \quad n = 0, 1.$

Lock points at $\phi_C - \phi_{ref} = n\pi/2$, $n = 0$ gives correct lock, $n = 2$ inverts the data, and $n = 1$ or 3 swaps the two bit streams and inverts one of them.

5. $P_{BE} = \dfrac{11}{12}\, Q\left(\sqrt{\dfrac{2E_b}{7N_0}}\right), \quad \dfrac{11}{7}$, approximately 0.5 dB.