3F4 Digital Modulation - Examples Paper 2

1. Show that, for M-level amplitude shift keyed (ASK) signals with amplitude states that are symmetric about zero, the probability of symbol error P_{SE} in an optimal receiver is given by:

$$P_{SE} = 2\left(1 - \frac{1}{M}\right)\ Q\left(\sqrt{\frac{6}{M^2-1}\ \frac{E_S}{N_0}}\right)$$

where E_S is the energy per symbol and N_0 is the noise power spectral density at the demodulator input. From this expression, derive the probability of bit error P_{BE} in terms of the energy per bit E_b .

Extend this result to the case of quadrature amplitude modulation (QAM) with M^2 levels, and show how your results may also apply to binary and quadrature phase shift keying (BPSK and QPSK) as special cases.

A signal at 10 Mb/s is received with a power of 10 pW, and the noise figure and temperature of the receiver are 13 dB and 20 C respectively. Calculate P_{BE} for the following modulation schemes: BPSK, QPSK, 4-ASK, 16-ASK and 16-QAM.

What are the implications of your results for practical bandlimited systems?

2. A Digital Audio Broadcast (DAB) signal employs QPSK modulation on a coded orthogonal frequency division multiplexed (COFDM) set of carriers. Estimate the maximum bit rate that would be available to the user for a system with the following parameters:

Channel bandwidth: 2.4 MHz
Carrier frequency spacing: 1000 Hz
Symbol rate on each carrier: 750 symbol s^{-1}
Error-correction code rate: 1:2

Why is QPSK modulation a good choice for COFDM systems designed for reception of good quality digital audio in vehicles.

Practical transmission channels exhibit multipath effects. Giving reasons, estimate the maximum path delay variation that could be tolerated by the above system before the bit error performance against noise starts to become degraded.

3. A digital TV system employs COFDM with 64-QAM as the underlying modulation method. A rate 2:3 convolutional code provides error correction and 10% of the OFDM carriers are used as pilot tones. Explain the purpose of the pilot tones.

If the signal bandwidth is 9 MHz, the guard band to allow for multipath is 10μs and the FFT demodulation block period is 200μs, estimate the number of TV channels that could be accomodated within this COFDM signal (assume that one TV channel requires a data rate of 4 Mb/s).

Compare the bandwidth efficiency (bits/s of user data rate per Hz of bandwidth) for this digital TV system with that of the DAB system in the previous question, and suggest reasons why the audio system has been designed to be less efficient.

Relevant past Tripos questions: 3F4 has been running for a number of years and virtually all past Tripos questions are relevant.

Nick Kingsbury, February 11, 2016.

Answers:

1. For BPSK and QPSK, $P_{BE} = 3.22 \; . \; 10^{-7}$

   For 4-ASK and 16-QAM, $P_{BE} = 6.17 \; . \; 10^{-4}$

   For 16-ASK, $P_{BE} = 0.0657$

2. Max bit rate for user $= 1.8 \; . \; 10^6$ b/s.

   Max path delay variation = 0.333 ms.

3. Max number of TV channels = 7 (this figure allows approx 10% overhead for the channel multiplexer).

   Bandwidth efficiency of TV system = 3.43 b/s/Hz.

   Bandwidth efficiency of DAB system = 0.75 b/s/Hz.