ENGINEERING TRIPOS PART IIB

Saturday 29 April 2006 9 to 10.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: 2

1 (a) We have an observed image $y(u_1, u_2)$ that has been captured with an imperfect imaging system. Assuming that y can be modelled as a linear distortion of the true image $x(u_1, u_2)$ plus additive noise $d(u_1, u_2)$, write down an expression relating the true image to the observed image in the continuous case, assuming that the distortion is represented by a point-spread function $h(u_1, u_2)$. Define all quantities used. [15%]

(b) Rewrite the expression obtained in part (a) in discrete form. If the distorting function is known and the noise is neglected, explain what is meant by *inverse filtering* and describe how this can be used to estimate the true image. [20%]

(c) Indicate the reasons for the poor performance of the inverse filter on real images. Describe how we can modify the inverse filter to produce the *pseudo-inverse filter* (also known as the *generalised inverse filter*) in order to improve performance. [15%]

(d) In the presence of significant noise the pseudo-inverse filter still performs poorly. The optimal *linear* filter for reconstruction is the *Wiener filter*. In what sense is the Wiener filter optimal?

Writing our true and observed images in vector form as $\mathbf{x}$ and $\mathbf{y}$ respectively, and assuming the Gaussian noise is $\mathbf{d}$ and the linear distortion is represented by matrix L, explain why we are able to write the probability of $\mathbf{x}$ given $\mathbf{y}$ as

$$P(\mathbf{x}|\mathbf{y}) \propto \exp\{-\tfrac{1}{2}[(\mathbf{y} - L\mathbf{x})^T N^{-1}(\mathbf{y} - L\mathbf{x}) + \mathbf{x}^T C^{-1}\mathbf{x}]\} \quad (1)$$

and describe the nature of the matrices N and C, noting any assumptions made. (Hint: Using Bayes theorem, $P(\mathbf{x}|\mathbf{y})$ is proportional to the product of the likelihood and the prior for $\mathbf{x}$). [35%]

(e) If $\mathbf{x} = W\mathbf{y}$ maximises (1), W is the Wiener filter matrix. Outline how, by taking alternative priors for $\mathbf{x}$, we are able to improve upon the Wiener filter. Give an example of one such alternative prior. [15%]

2 (a) A commonly used technique for processing images is *histogram equalisation*.

(i) Explain, qualitatively, the concept of histogram equalisation, and describe when it would be useful. [15%]

(ii) Assuming that we have a range of greyscale values from 1 to 8, consider the 4×4 image given in Fig. 1. Sketch and comment on the histogram of this image. [10%]

(iii) Perform histogram equalisation on this image by finding the set of transformed values $\{y_k\}$, $k = 1, .., 8$, onto which the original greylevels are to be mapped. Sketch the new equalised image and its histogram, commenting on how well the process has worked. [25%]

(b) An image $g(u_1, u_2)$ consists of a single white diamond with vertices at $(\pm a_1, 0)$ and $(0, \pm a_2)$, as shown in Fig. 2. Assume 'white' is assigned a constant value A in the shaded region of the figure, and the surrounding unshaded region is 'black' which is assigned zero.

(i) By considering a transformation of coordinates or otherwise, evaluate the Fourier transform of this image. [35%]

(ii) Estimate the bandwidths of the image stating carefully the measure and axes used. Although the image is not truly bandlimited, by taking these bandwidths as measures of the extent of the frequency ranges, describe how this image should be sampled to avoid aliasing. [15%]

1	1	2	2
3	2	4	3
2	4	3	2
2	3	1	1

Fig. 1

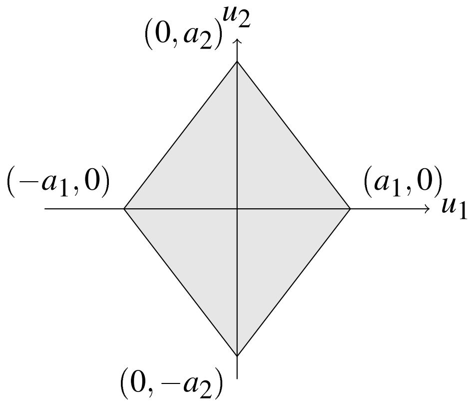

Fig. 2

3 (a) The 2-D Haar transform of a 2×2 pixel image patch, represented by the matrix

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

is given by

$$Y = \frac{1}{2} \begin{bmatrix} a+b+c+d & a-b+c-d \\ a+b-c-d & a-b-c+d \end{bmatrix}$$

Show how this transformation can be represented in the form $Y = T\,X\,T^T$ and calculate the 2×2 matrix T. In addition, discuss why the Haar transform is a simple but effective first stage of an image compression system. [30%]

(b) Show that T is an orthonormal matrix, and explain why this is an important property in a transform which is to be used for image compression. [20%]

(c) To improve the effectiveness of the transform, T is increased to a 4×4 matrix, of the form

$$T = \begin{bmatrix} p & p & p & p \\ q & r & -r & -q \\ p & -p & -p & p \\ r & -q & q & -r \end{bmatrix}$$

Calculate the positive parameters p, q and r if T is to remain orthonormal and $r/q = \tan(\pi/8)$. Give the name of this new transform and suggest why this particular ratio of r/q was specified. [30%]

(d) The standard JPEG image coding algorithm employs a transform matrix which is 8×8 in size. Discuss the main tradeoffs which led to this choice of size for JPEG. How do wavelet-based compression schemes overcome the need for such tradeoffs to be made? [20%]

4 (a) A 2-band analysis filter bank comprises a lowpass and highpass pair of filters, $H_0(z)$ and $H_1(z)$, in parallel, and each is followed by a 2:1 downsampling operation. Sketch this arrangement and the corresponding arrangement that is used in the 2-band reconstruction filter bank which contains filters $G_0(z)$ and $G_1(z)$. Briefly explain how such filter banks may be arranged to produce a two-dimensional wavelet transform. [25%]

(b) Show that the conditions required for perfect reconstruction from a 2-band analysis and reconstruction filter-bank system are given by

$$G_0(z)\,H_0(z) + G_1(z)\,H_1(z) = 2 \qquad \text{and} \qquad G_0(z)\,H_0(-z) + G_1(z)\,H_1(-z) = 0$$

If we define $G_1(z) = zH_0(-z)$, find a corresponding expression for $G_0(z)$ and derive constraints on coefficients of the even-order terms of the function $P(z) = H_0(z)\,G_0(z)$ in order to ensure perfect reconstruction. Why are there no constraints on the coefficients of the odd-order terms of $P(z)$? [30%]

(c) If we choose a transformation $Z = \frac{1}{2}(z + z^{-1})$ and express $P(z)$ as $P_t(Z)$ which is a polynomial in a finite number of non-negative powers of Z, what additional constraint does this place on $P(z)$ and why is this desirable for image coding applications? Show also how the constraints from part (b) apply to $P_t(Z)$. [15%]

(d) If $P_t(Z)$ is of the form

$$P_t(Z) = (1 + Z)^3\, R(Z)$$

show that the $R(Z)$ with the lowest complexity to satisfy the constraints for perfect reconstruction is of the form $(a + bZ + cZ^2)$ and calculate a, b and c. [15%]

(e) Explain how $H_0(z)$ and $G_0(z)$ may be derived from $P_t(Z)$ (without actually doing so), and what steps should be taken in this process in order to produce a wavelet image coder with good visual performance. [15%]

END OF PAPER